Ibn al-Haytham's Analysis Of the Moon Illusion

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Let me start with a very brief biographical sketch of Ibn al-Haytham.

Abū ʿAlī al-Ḥasan bin al-Ḥasan bin al-Haytham

أبو علي الحسن بن الحسن بن الهيثم

(965?-1040?)

Unfortunately, what we know, or think we know, about his life and works is based almost entirely on two thirteenth-century sources, both of them written long after his death and both of them problematic. One, for instance refers to Ibn al-Haytham by the given name "al-Hasan," the other by the given name "Muhammed." And there are other inconsistencies as well, some of them significant. Consequently, it's difficult to establish the facts of his life with any certainty. The best we can do is proceed with caution.

For a start, there's general agreement that Ibn al-Haytham was born in Basra, Iraq, sometime around 965 and died in Cairo around the very beginning of 1040. A gifted mathematician with an unusual flair for clear, systematic thinking, he wrote on a wide variety of technical subjects from pure geometry and arithmetic to optics, astronomy, and cosmology. All told, he's credited with nearly 100 works on scientific subjects. A little more than half are currently extant in complete or partial form.

By far the longest and most influential of these works was the monumental…

Kitāb al-Manāẓir
كتاب المناظر
"Book of Optics"

…*Kitāb al-Manāẓir*, or "Book of Optics," which he likely composed in his later years. One of only two of his writings to have been translated into Latin during the Middle Ages, it appeared in that form under the title…

Kitāb al-Manāẓir
كتاب المناظر
"Book of Optics"

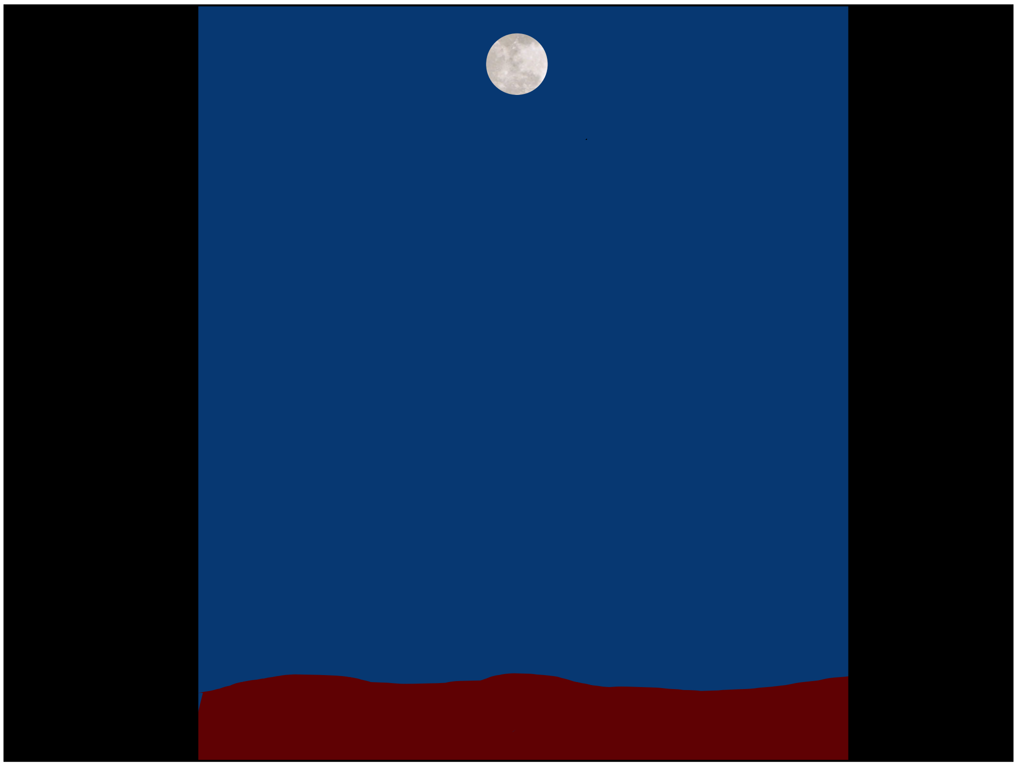
De aspectibus
"On Visual Appearances"
al-Ḥasan = Alhacen

…*De aspectibus*, or "On Visual Appearances," and was attributed to Alhacen, a fairly accurate Latin transliteration of Ibn al-Haytham's given name. It's in this version, not the Arabic original, that the *Kitāb al-Manāẓir* gained widespread acceptance as a canonical source in optics during the later Middle Ages and Renaissance. It's also this version that I spent most of my academic career editing and translating.

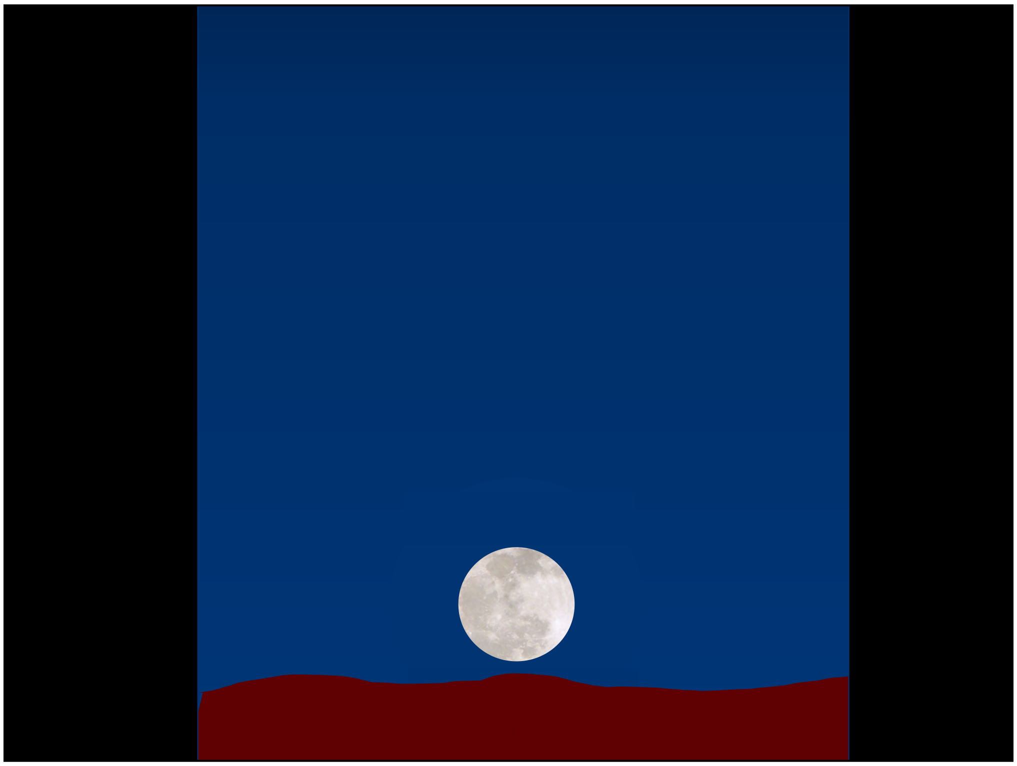
Consisting of seven books, the *Kitāb al-Manāẓir* covers all aspects of vision from the physics of light to the physiology and psychology of visual perception. The resulting account, much of it supported by complex geometrical reasoning, is remarkably broad in scope, detailed in analysis, and sophisticated in approach—so much so that on its basis Ibn al-Haytham has earned a place among the most prominent scientific thinkers not just of his day but of all time.

It is therefore upon the *Kitāb al-Manāẓir* that I will focus in this brief talk. More specifically, I will focus on Ibn al-Haytham's account of the Moon Illusion at the very end of that treatise. I've chosen to focus on this account for two reasons: first, because it requires no technical expertise to understand or appreciate, and second, because it exemplifies the ingenuity with which Ibn al-Haytham adapted, assimilated, and eventually transcended his sources.

Briefly put, the Moon Illusion is based on the apparent magnification of the Moon or Sun as they descend toward the horizon. This is illustrated here,…



…where the Moon looks fairly small when seen high in the evening or night sky and significantly larger…



…when viewed near the horizon. We've all experienced this phenomenon, and when asked to estimate how much larger the Moon or Sun looks at horizon, many of us guess that they are at least twice as large, even larger in some cases.

In fact, no such magnification takes place. The Moon and Sun are no bigger at horizon than directly overhead at zenith, both of them subtending an arc of about half a degree at the two points. Both celestial bodies, in short, subtend the same visual angle no matter where they are in the sky. At times, in fact, they can appear flattened, and thus *shrunken*, by atmospheric refraction at the horizon, as illustrated here,…

…where the moon's shape is distorted by such flattening.

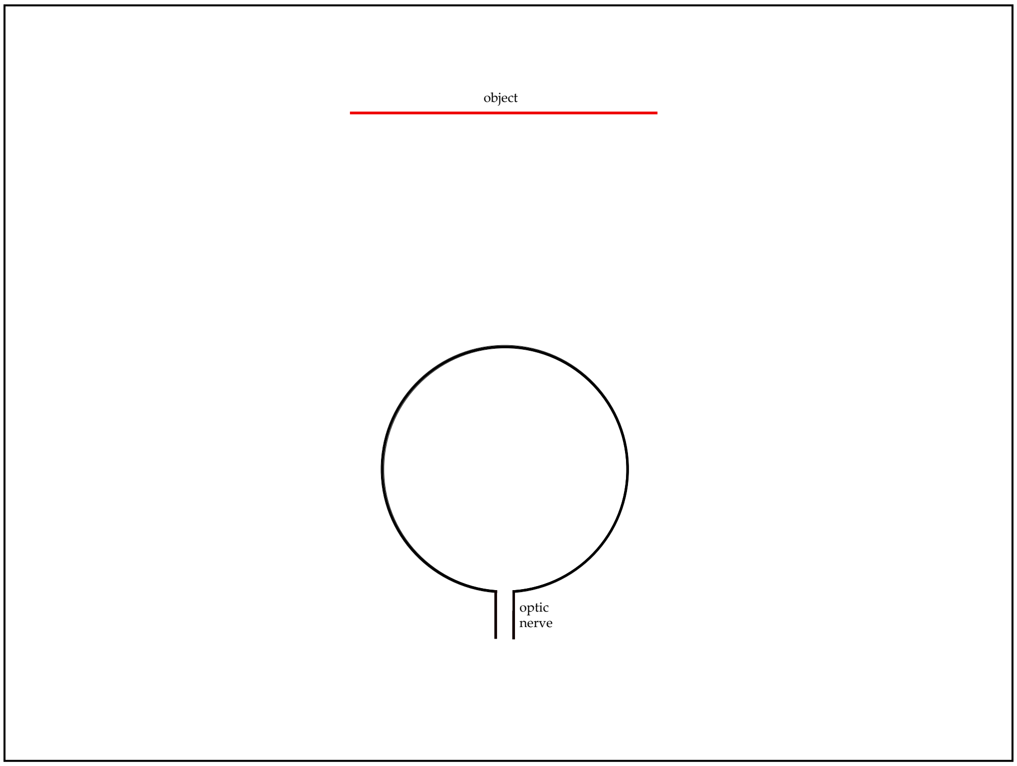
Here we see the same effect with the sun, the result being that, like the moon, it appears somewhat oblate at the horizon.

The problem, therefore, is to explain why the *apparent* enlargement occurs at horizon, and, as we'll see, Ibn al-Haytham's solution depends on a rather sophisticated understanding of how we perceive distance and size. But in order to appreciate the elegance of that solution, we need to take a brief look at his key source: the Greco-Roman thinker Ptolemy,…

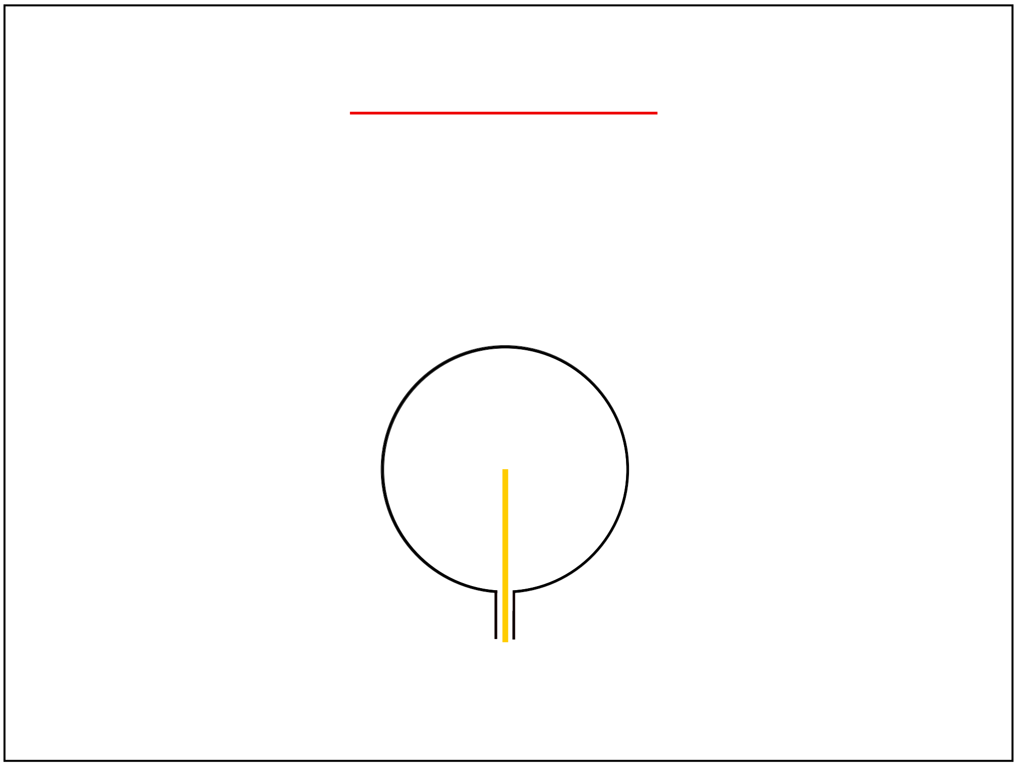
Ptolemy
Baṭlumyūs
بطلميوس
Optics (160 CE)

…whose *Optics* served as a template for the *Kitāb al-Manāẓir*.

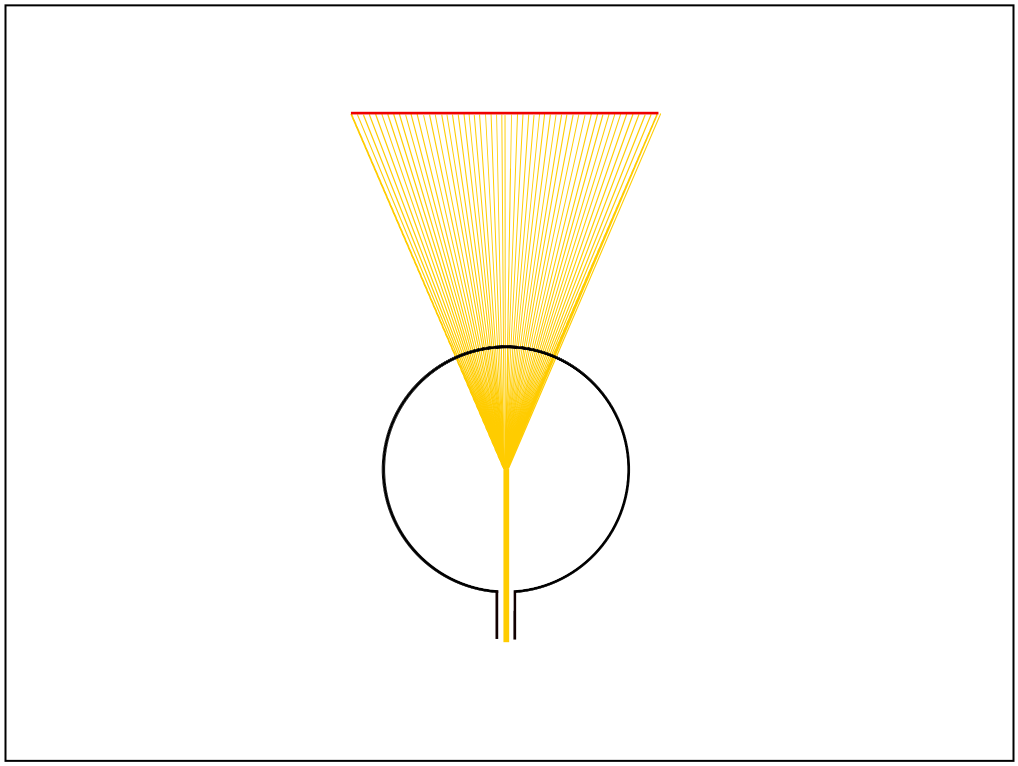
Key to Ptolemy's account of vision is the assumption that the eye, represented in this crude schematic,…



…emits a sort of luminous flux passed into it through the hollow optic nerve from the brain**.**

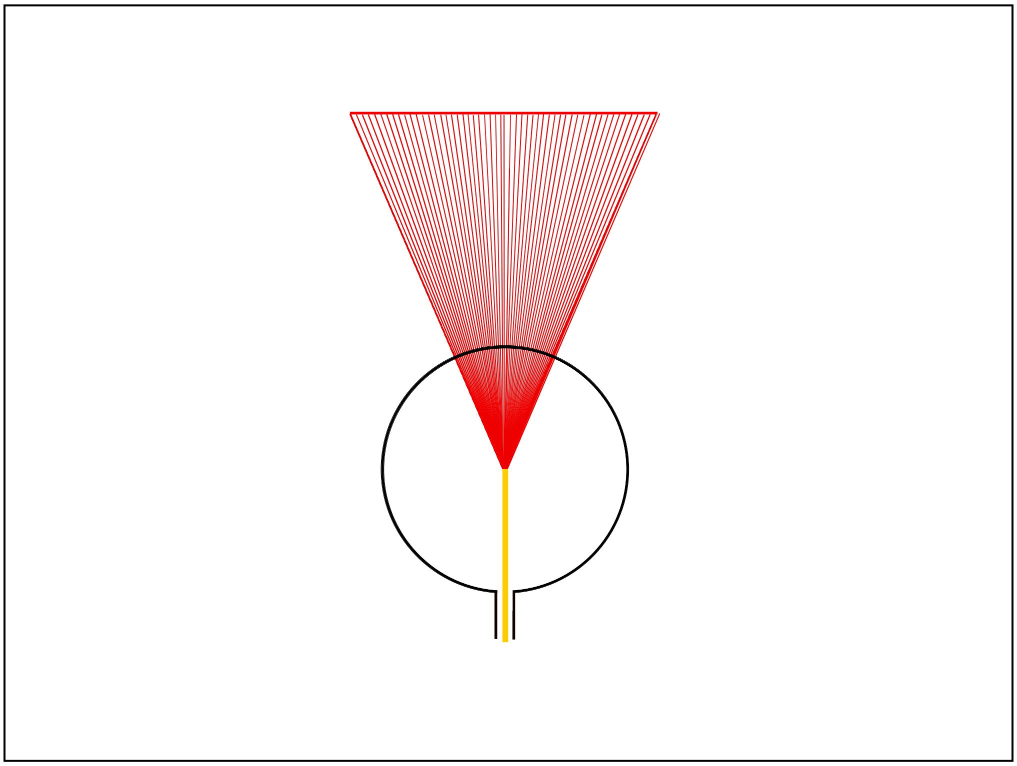


That flux is then radiated out along perfectly straight lines toward external objects,…

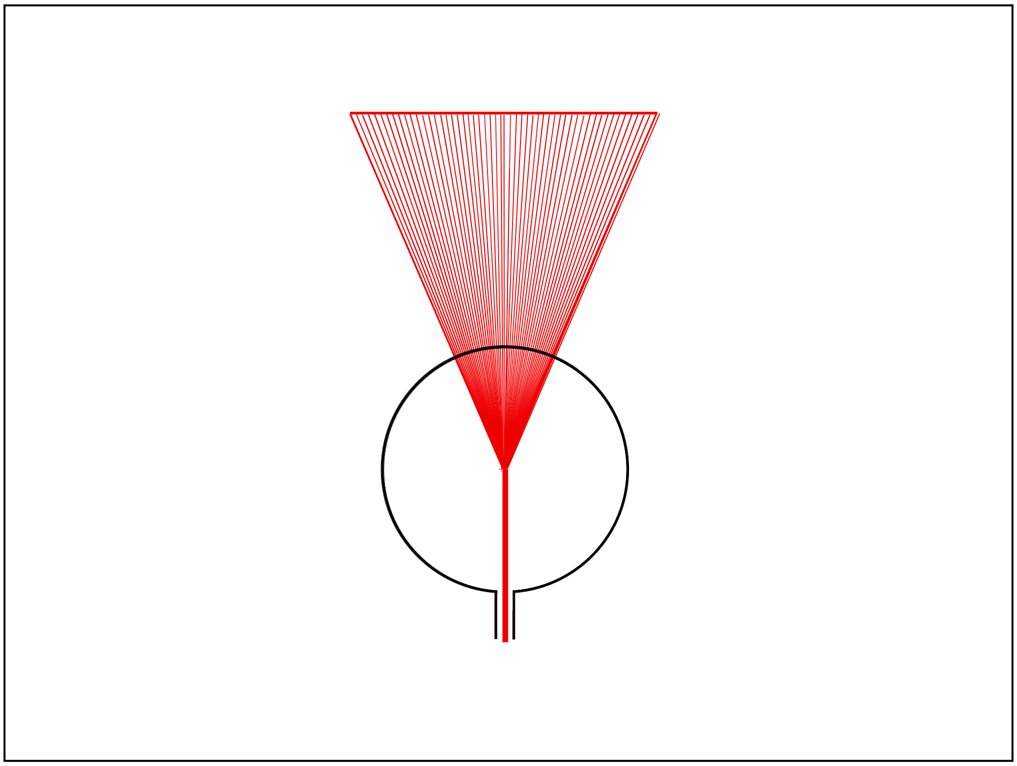


…such as the one represented by the thick red line. Taken as a whole, these radial lines form a cone bounded by the pupil, with its vertex in the center of the eye and its base defining the field of view. When they come into contact with visible objects in that field, the radial lines of flux “feel” them in a visual way.

The resulting visual impression, which is grounded in the object’s color, radiates back through those lines to the eye…

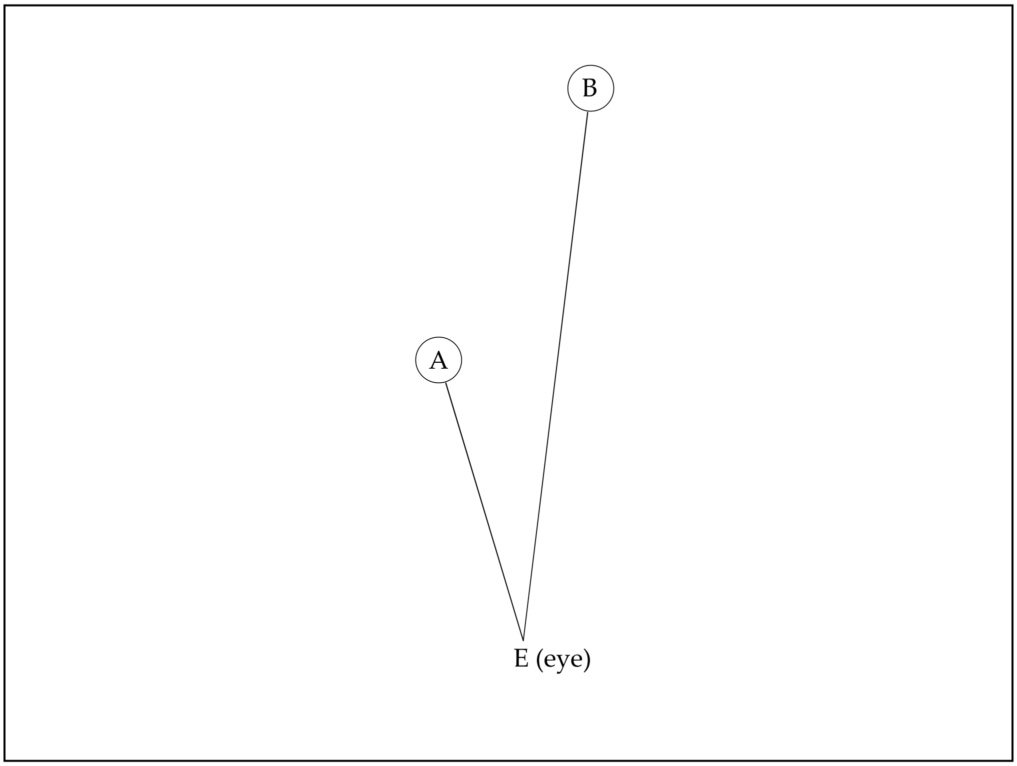


…in the form of a sensible representation. This representation is then passed back through the optic nerve to the brain,…



…where we make perceptual and intellectual sense of it. If, for instance, the lines of flux within the visual cone make contact with a horse, the impression arising from this contact is passed back to the eye in the form of an image loaded with implicit information that includes such things as its color, shape, location, size, and so forth. By perceptually and intellectually processing this information, we eventually realize that what we have seen is a horse and not a mule or an ox.

In order to make full visual sense of what we see, we need to know how far away it lies and how large it is. According to Ptolemy, we determine the distance of things through an innate sense of how long the lines of luminous flux between us and those things are. Accordingly,…

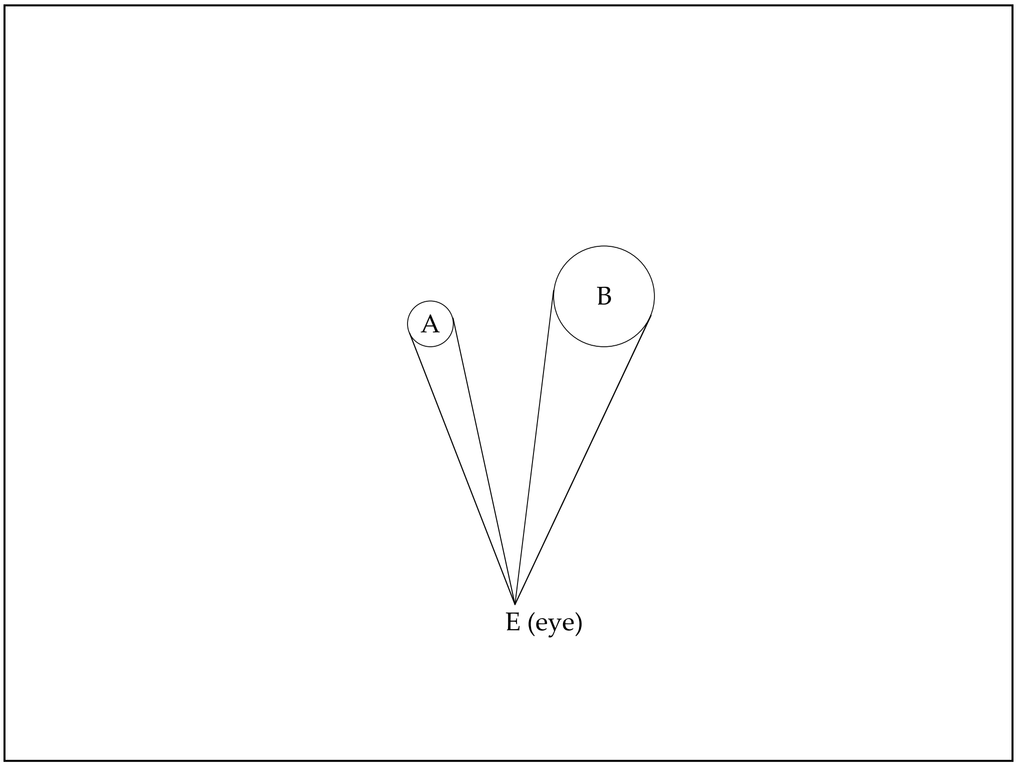


…we perceive that object B in this diagram is twice as far away as object A because we sense that line of flux EB is twice as long as line of flux EA.

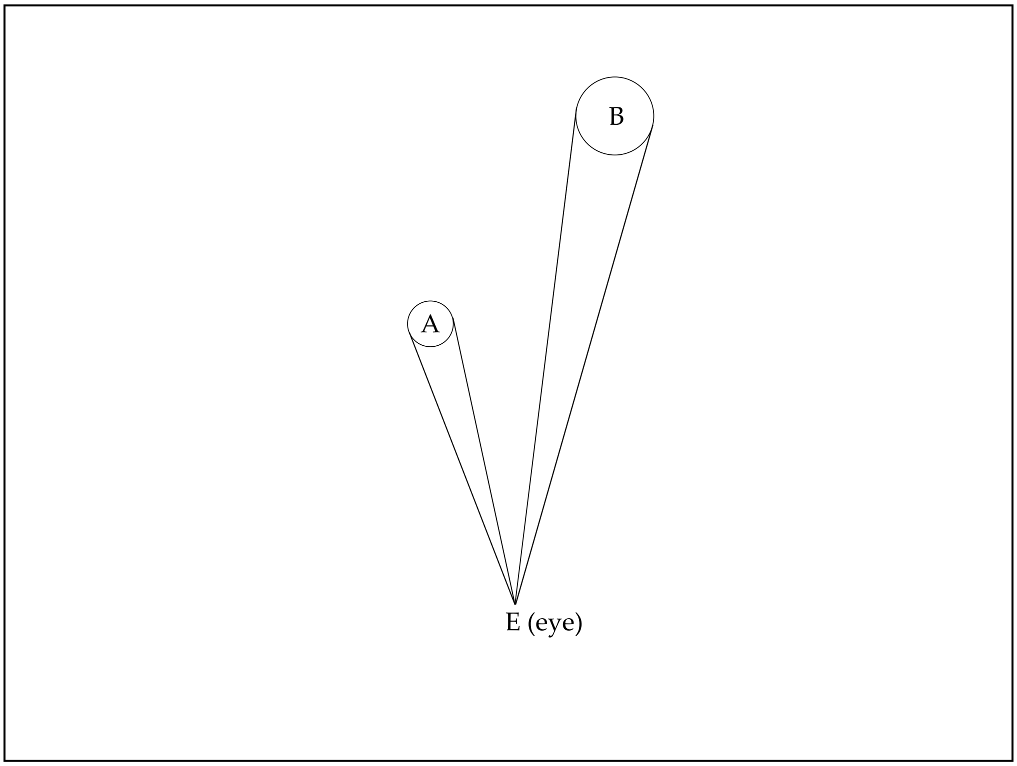
The visual perception of distance is therefore immediate and intuitive for Ptolemy, who invites us to think of it by analogy to how we determine distance with our arms, hands, and fingers. Just as we judge how far away things are according to how far we have to reach out to touch them, so our luminous flux tells us how far away things lie according to how far it has to reach out in order to make visual contact with them.

Likewise, just as our fingers and hands apprehend certain physical characteristics of things they touch—e.g., what shape they are, whether they are smooth or rough, whether they are hard or soft, and so forth—so the luminous flux informs us about various physical characteristics of the objects it touches, such things as color, texture, shape, and so forth.

Determining distance is also crucial to ascertaining the size of things, which is ultimately based on the visual angle subtended by them. For instance,…

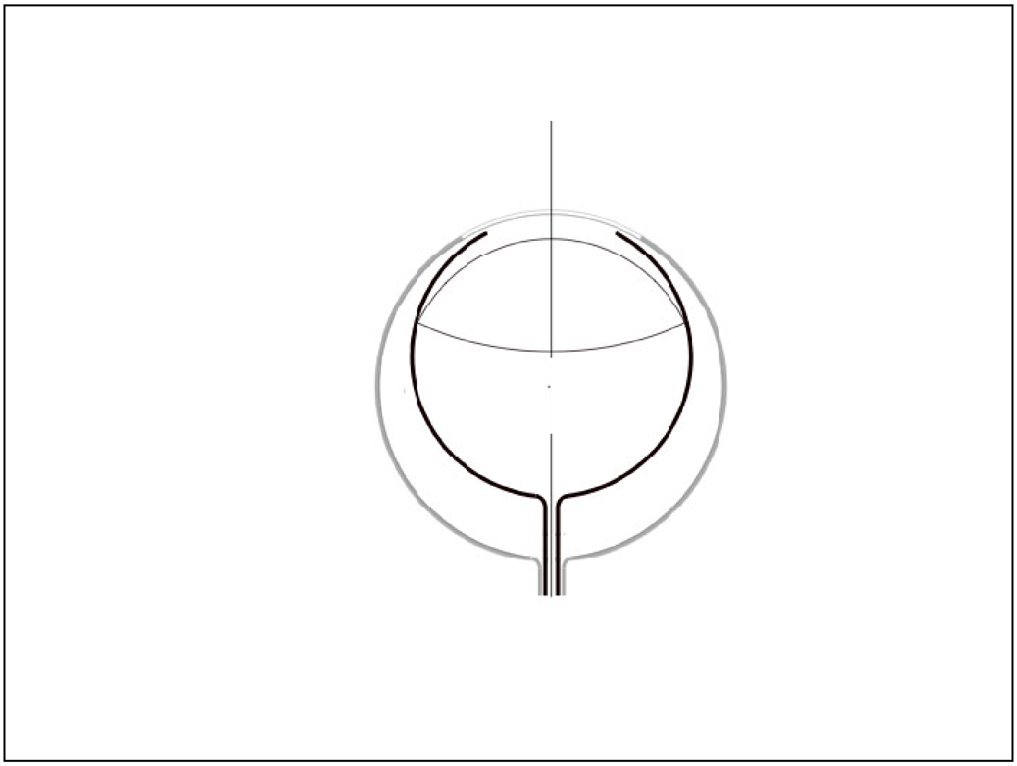


…if two objects A and B lie the same distance from the eye but subtend different visual angles, the one that subtends the larger angle will look larger, object B in this case. On the other hand, if the two objects lie at different distances from the eye, then the visual angle alone will be insufficient for determining size. That angle has to be correlated with distance.

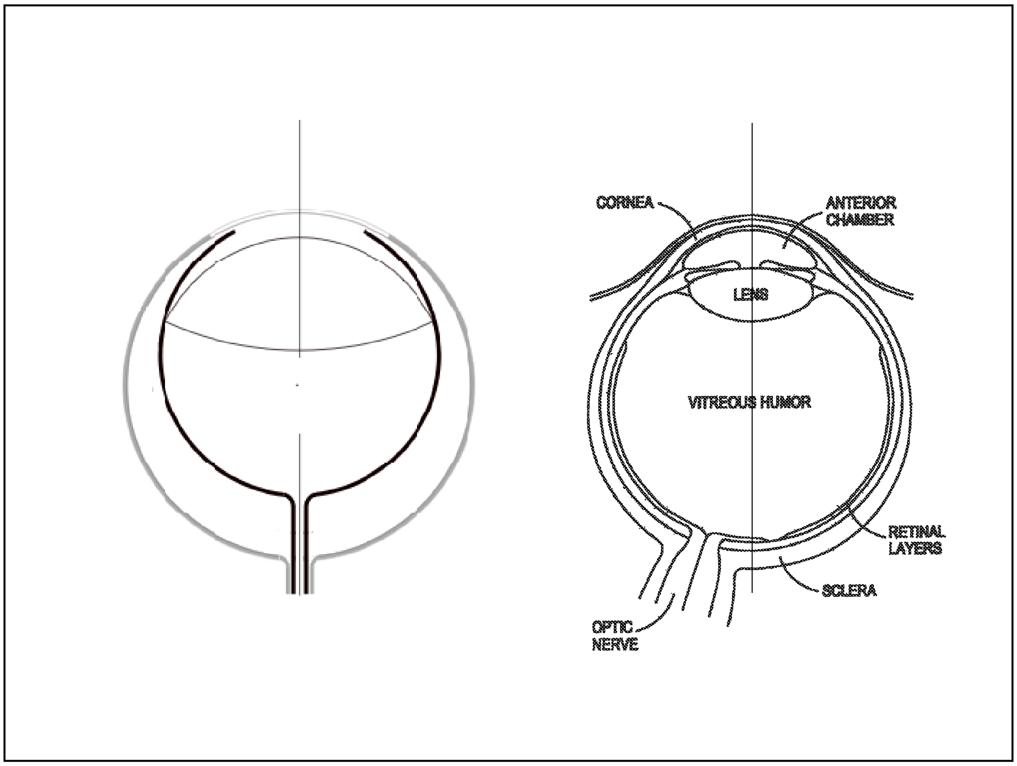


This is clear in the case of two objects A and B that lie different distances from the eye but subtend equal visual angles. The one that lies farther away, namely object B, will look larger to us because we sense that it lies farther away.

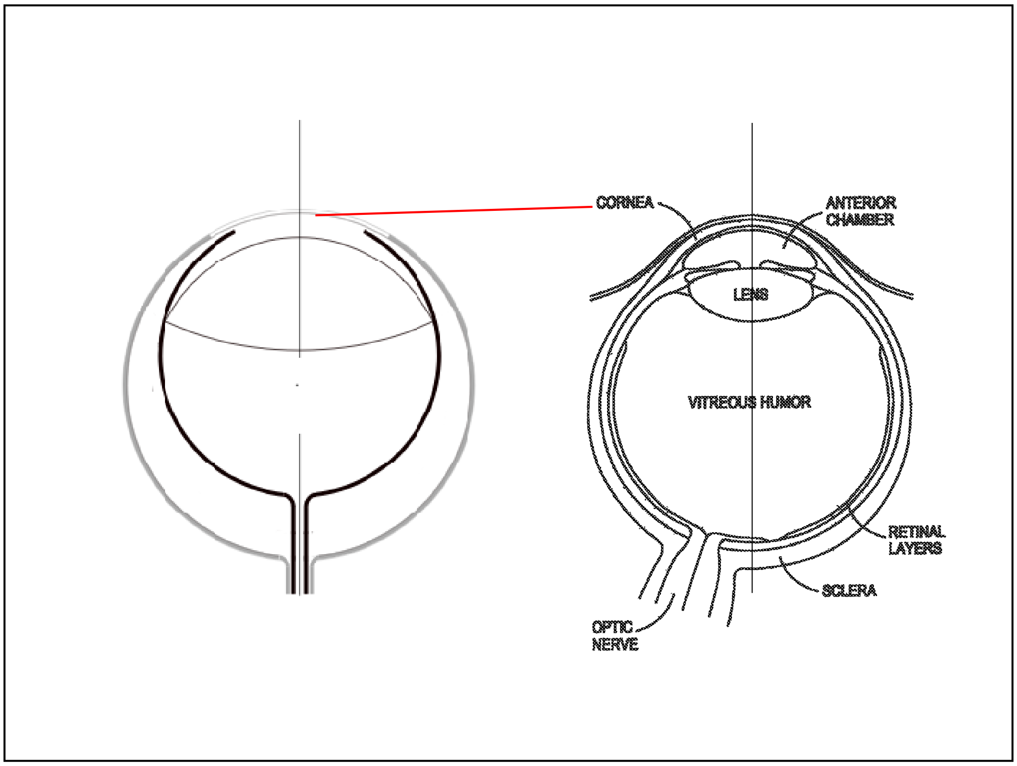
The virtue of this account of distance- and size-perception is its simplicity and its appeal to common sense. Most of us, I think, regard such perception as intuitive and immediate, as occurring automatically the instant we see something. Ibn al-Haytham, on the other hand, was forced to explain it in a more complex and sophisticated way because, unlike Ptolemy, who based his account of sight on luminous flux radiated out *from* the eye to visible objects, Ibn al-Haytham based his on the radiation of light from visible objects in *to* the eye. In assimilating Ptolemy's ray-theory, therefore, Ibn al-Haytham had to adapt it to his light-based theory of vision. So let's take a brief look at that theory, starting with his anatomical model of the eye.



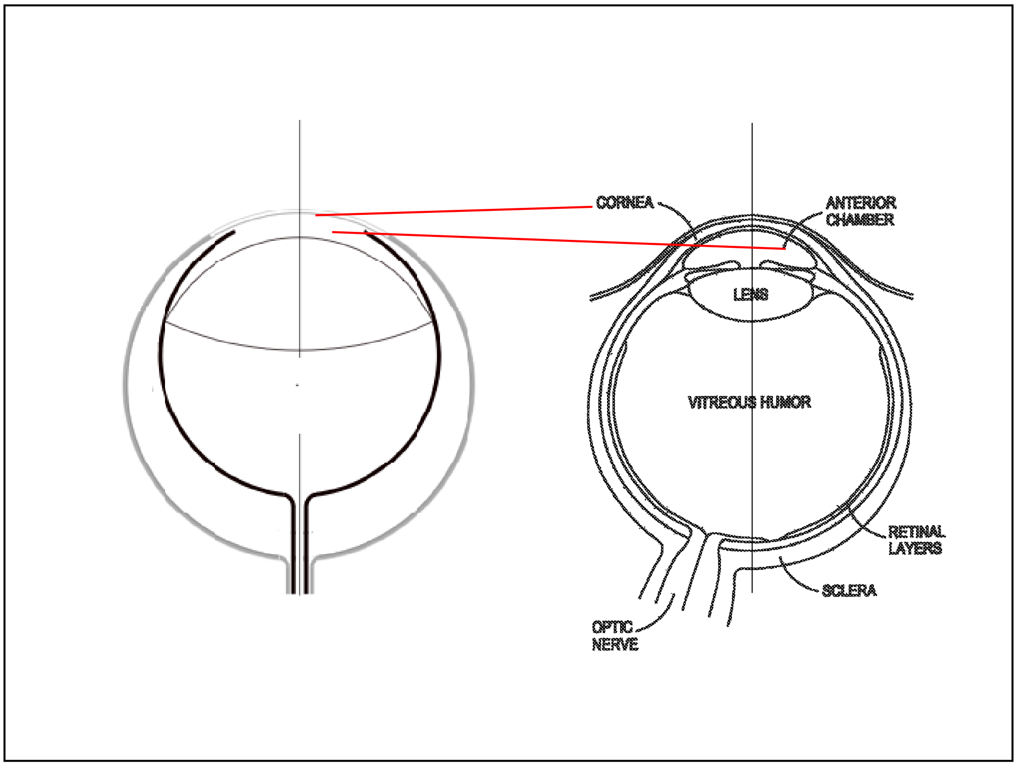
Here is a rough schematic of that model, and here…



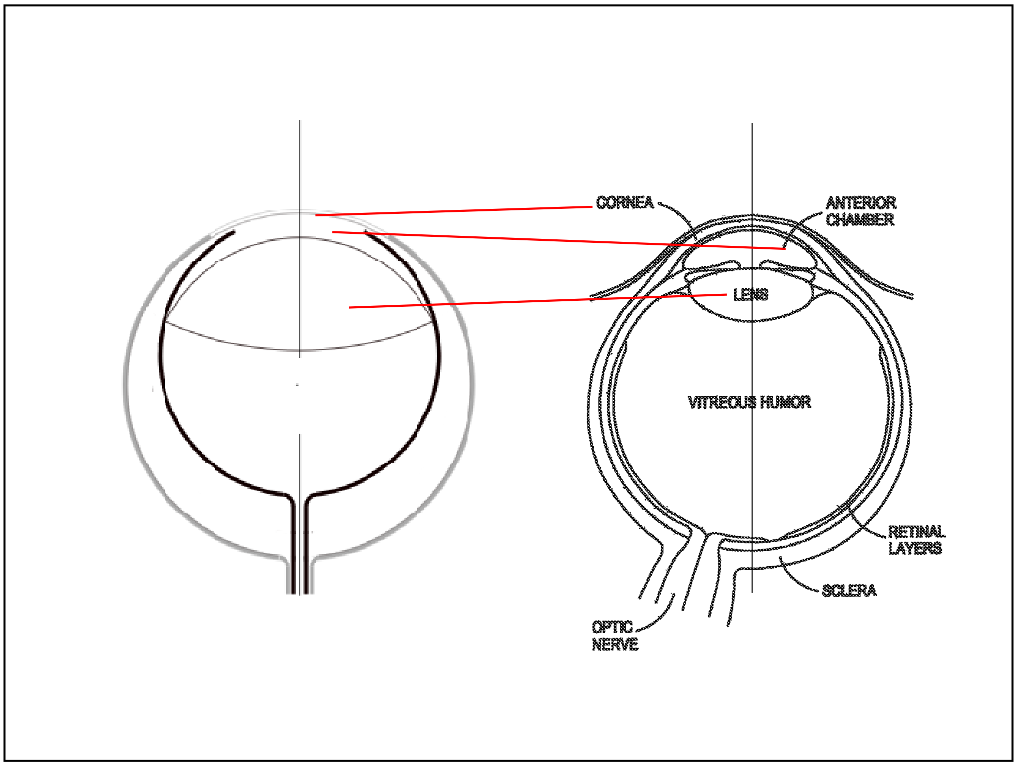
…it is beside a schematic of the eye according to current anatomical knowledge. As you can see, the two models are roughly similar in that they share the same basic components, that is, the transparent cornea…



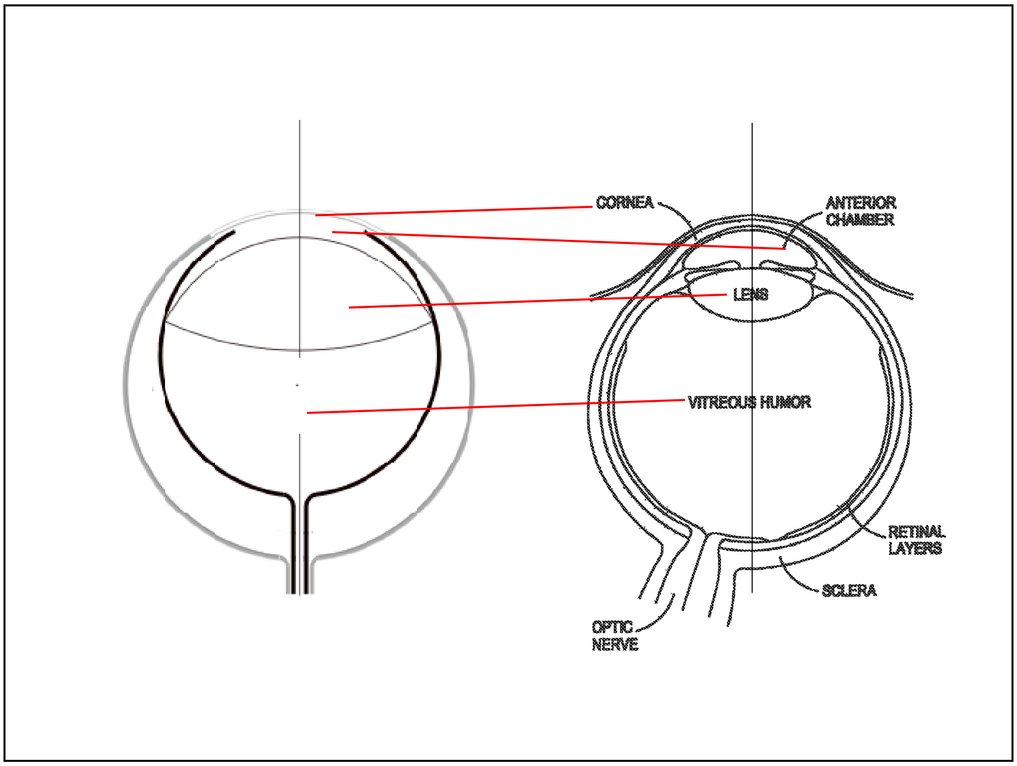
…at front, the anterior chamber…



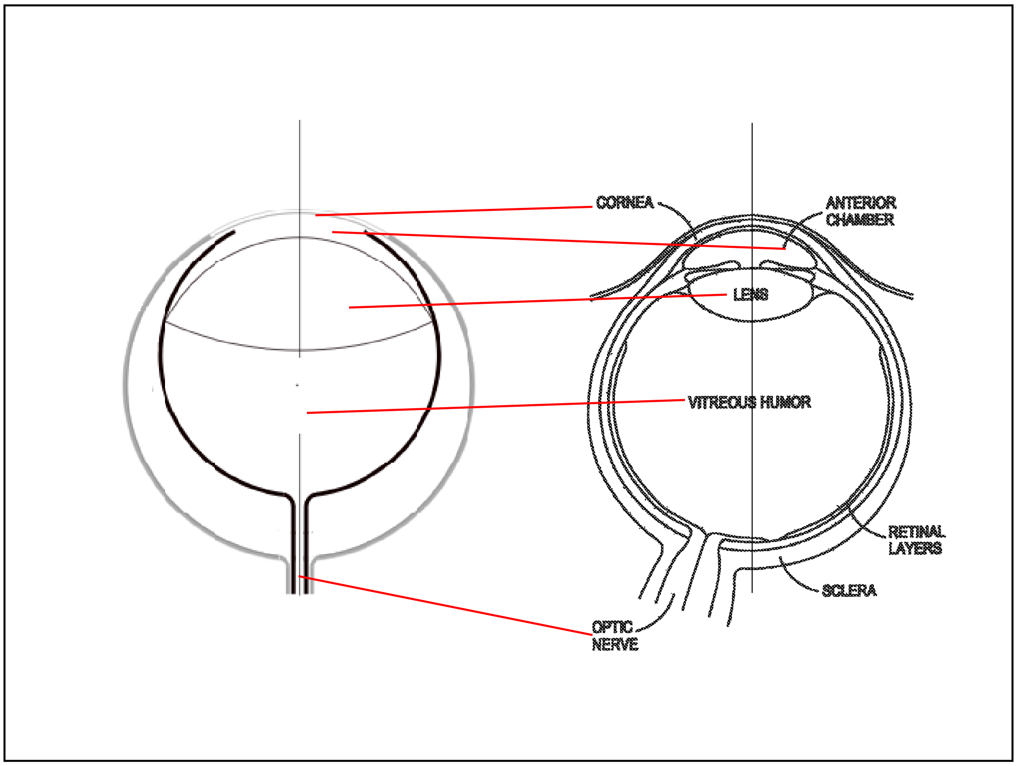
…between the cornea and the front of the lens, the lens itself,…



…the space behind the lens filled with vitreous humor,…



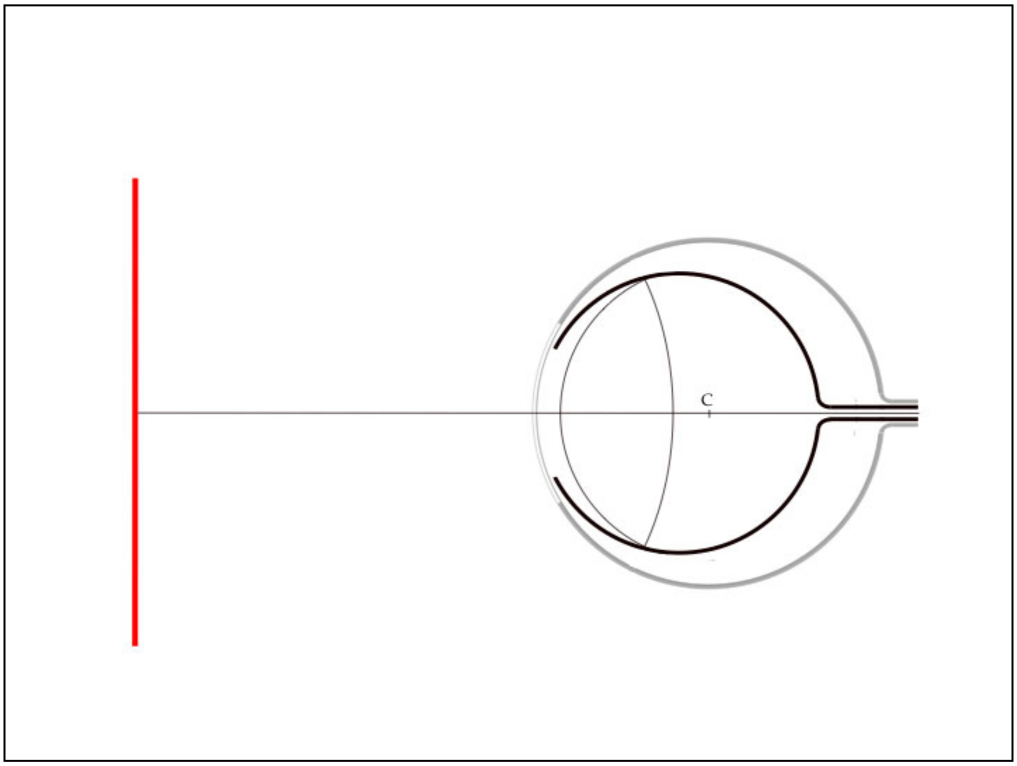
…and the optic nerve…



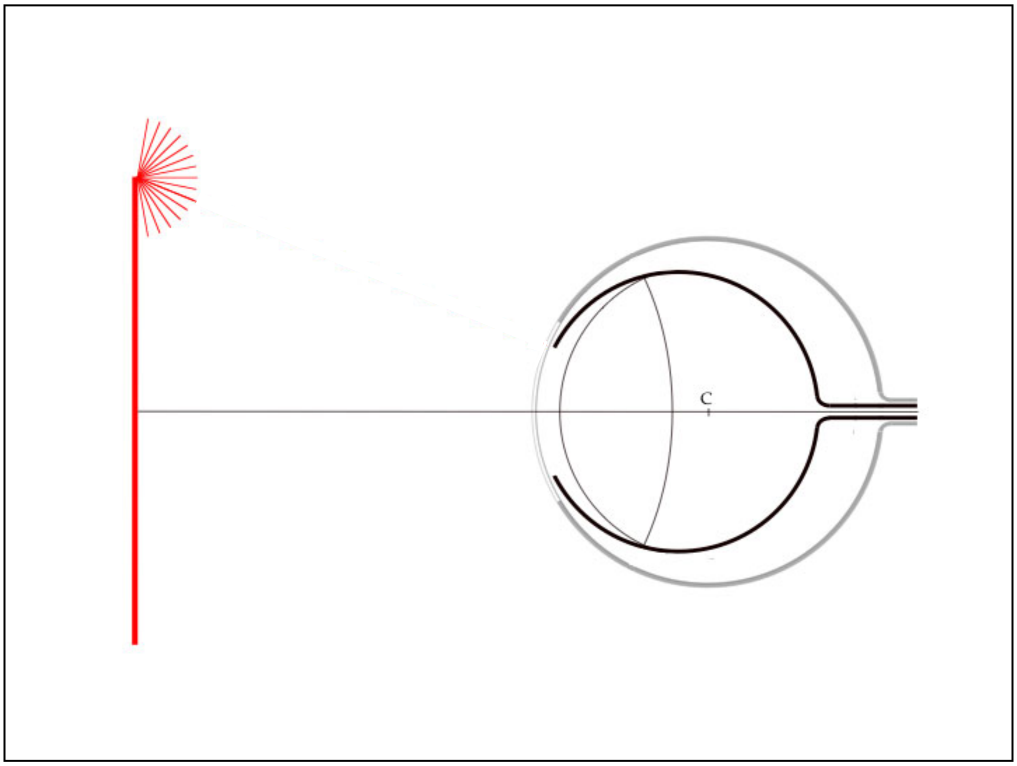
…at the back of the eye. Contrary to modern anatomical theory, Ibn al-Haytham followed the ancient Galenic medical tradition in assuming that the optic nerve is hollow.

You can also see that the geometry of Ibn al-Haytham's eye is somewhat different from that of its modern counterpart. In his model, unlike the modern one, the front surface of the lens is perfectly concentric with the cornea, which in turn is perfectly concentric with the eye as a whole. Also, in Ibn al-Haytham's model, the optic nerve lies directly along the eye's axis, whereas it is offset from that axis in the current model.

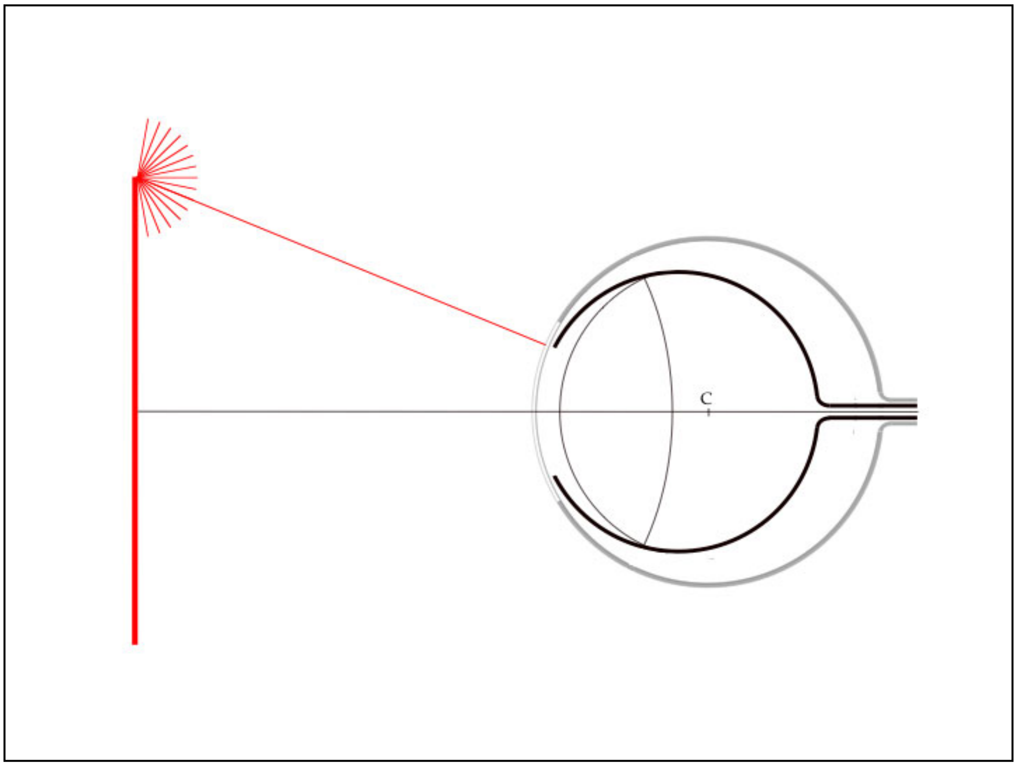
Now let's place an object in front of Ibn al-Haytham's eye,…



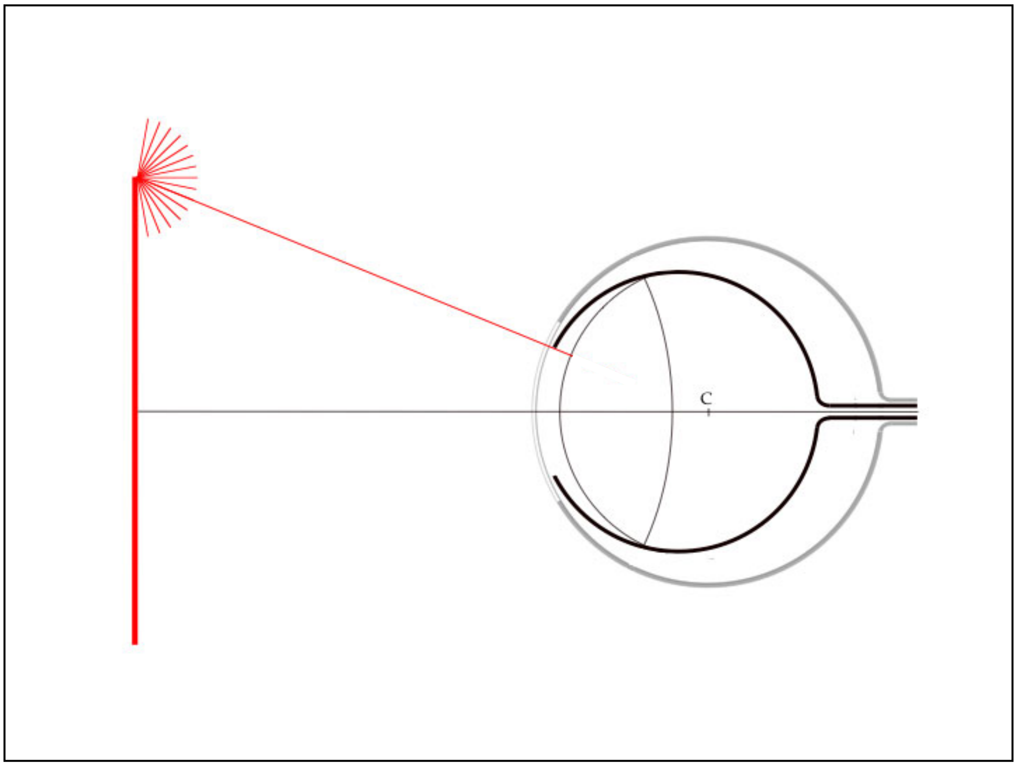
…as represented here by the thick red line. According to Ibn al-Haytham, every point on that object radiates its light—or, rather, its luminous color—in every possible direction, as represented here,…



…where the top endpoint of the object sends rays out toward every point in the surrounding space that's open to it. Of all those rays, only one…

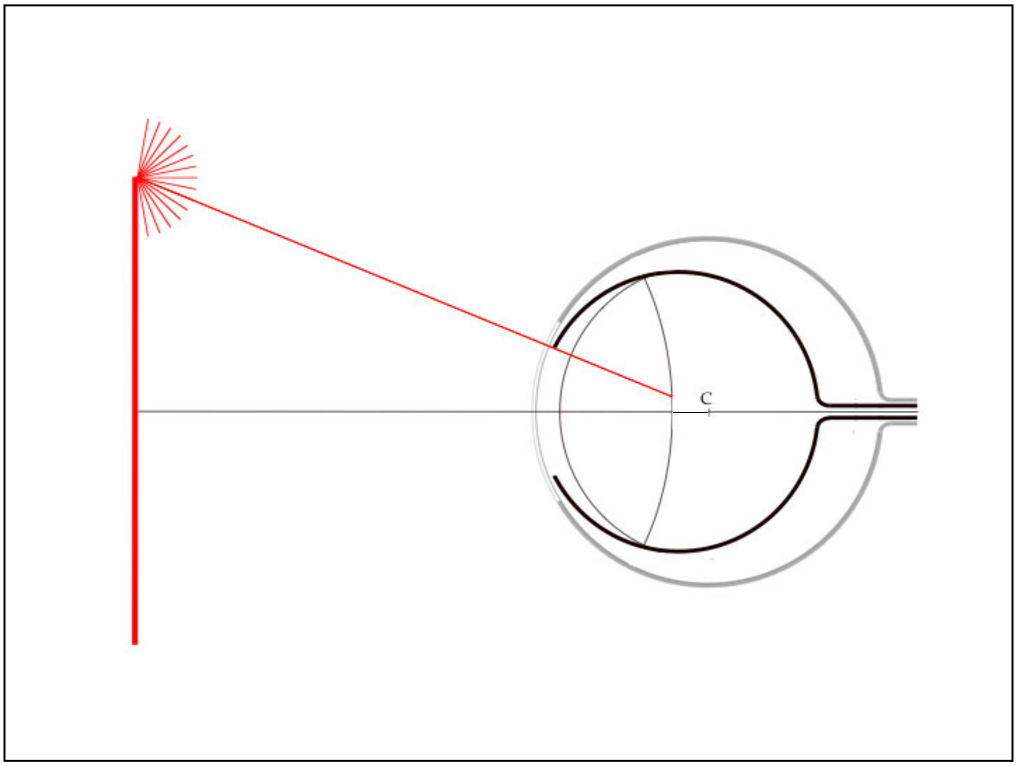


…reaches the front surface of the cornea along the perpendicular, so when it passes straight through,..

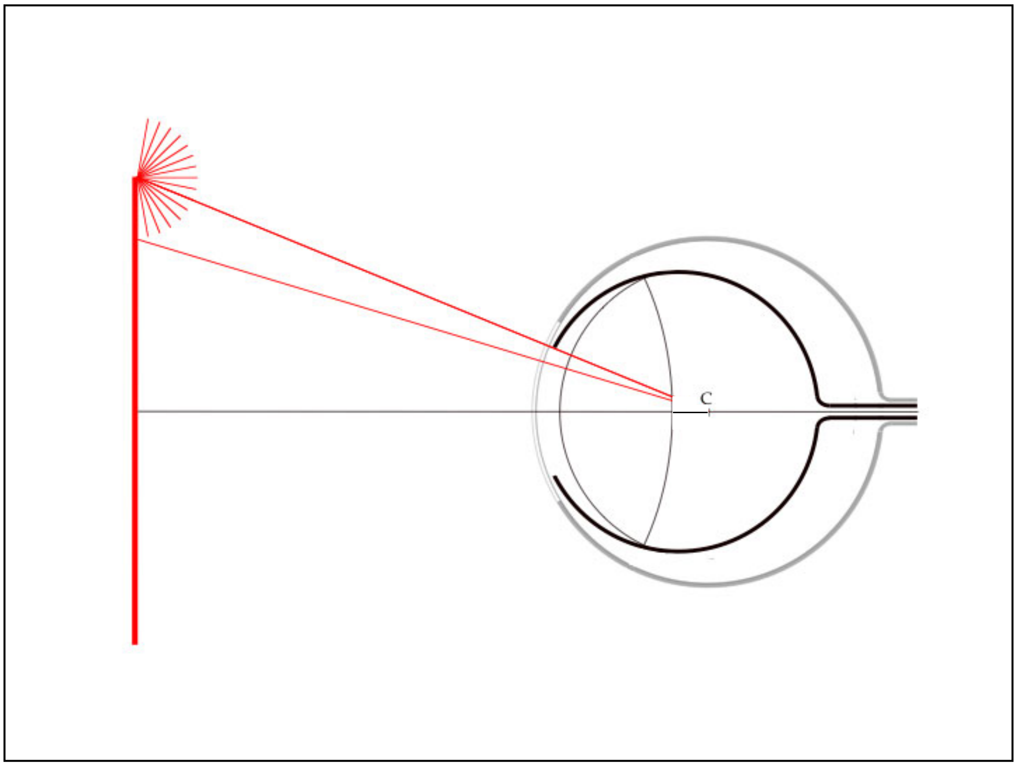


…it strikes the front surface of the lens along the perpendicular as well. This is dictated by the geometrical structure of the eye as Ibn al-Haytham describes it.

Being visually sensitive, the front surface of the lens feels this impinging ray in a way analogous to how we feel a small object striking our skin. It only feels those rays that strike it along the perpendicular, though, because they strike it most forcefully. The rest, being oblique, are too weak to be sensed. The resulting visual impression…



…then passes straight through the lens toward the center of the eye in conformity with the eye's geometrical structure. The same holds for the radiation from these spots on the object: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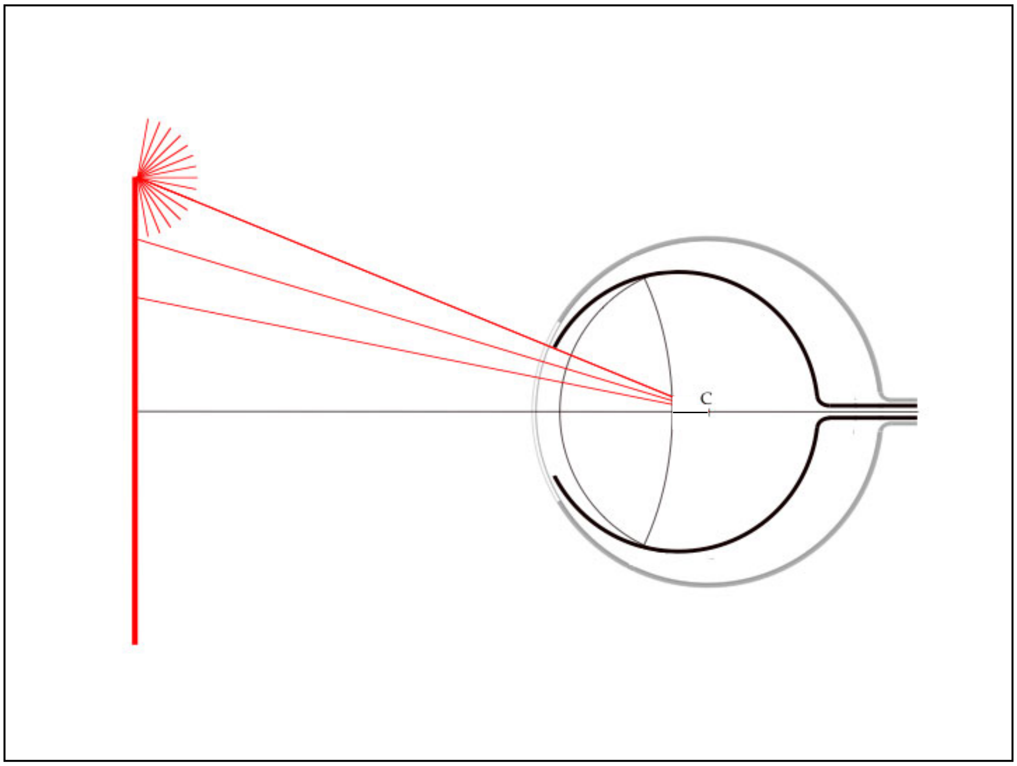
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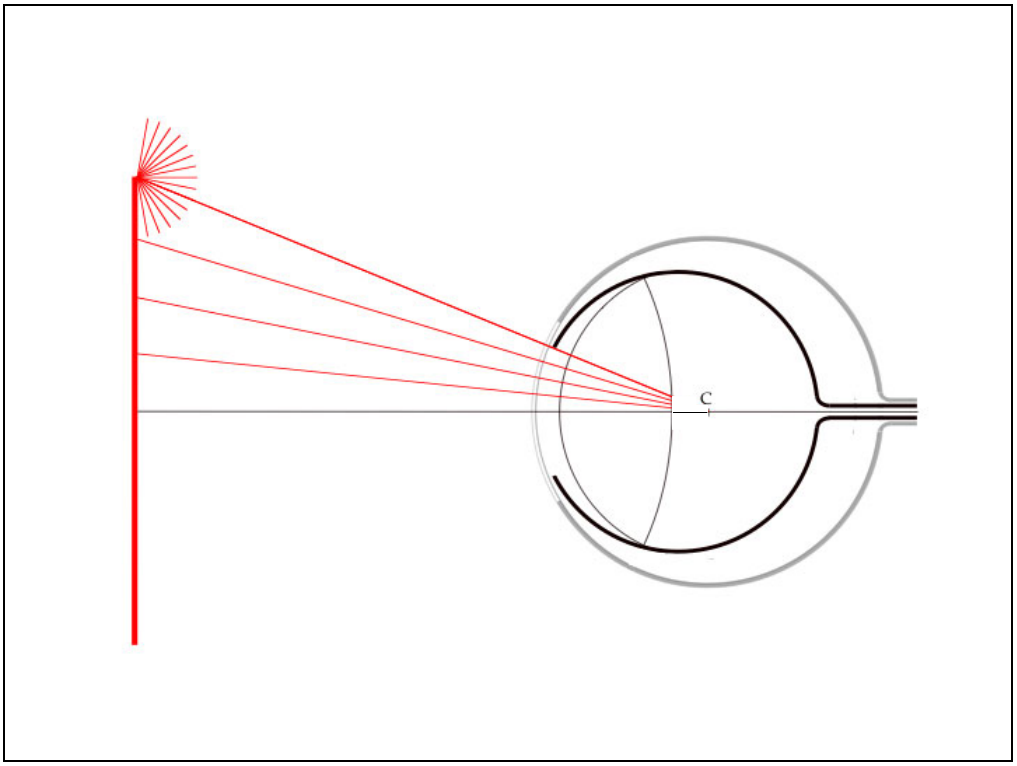
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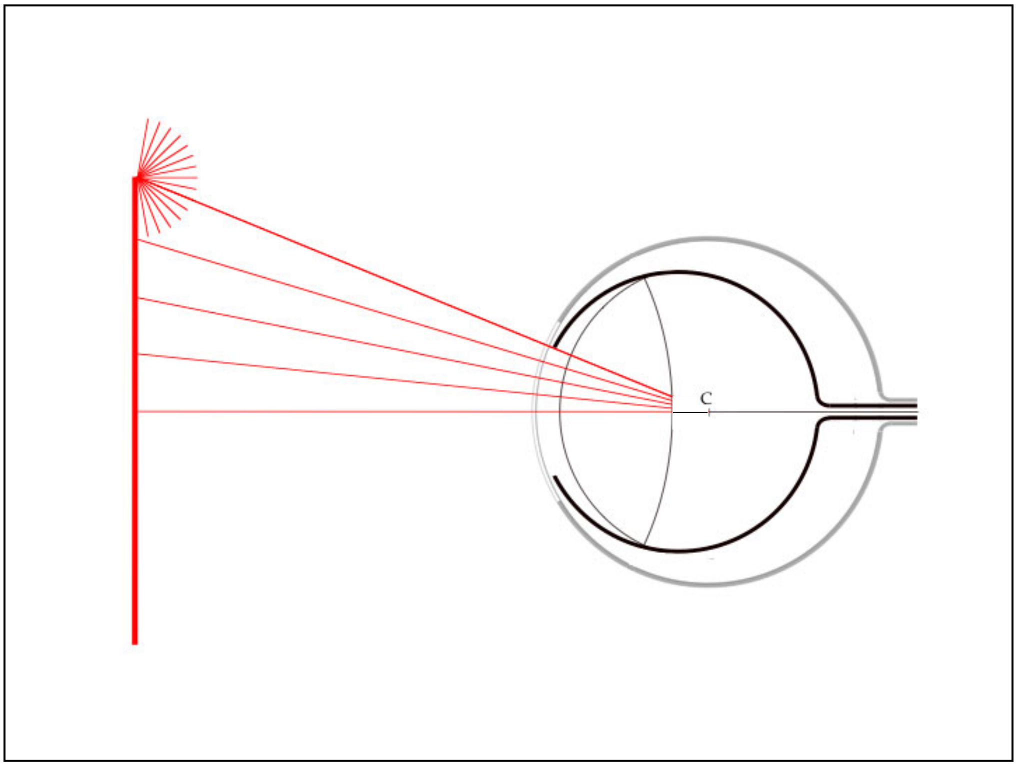
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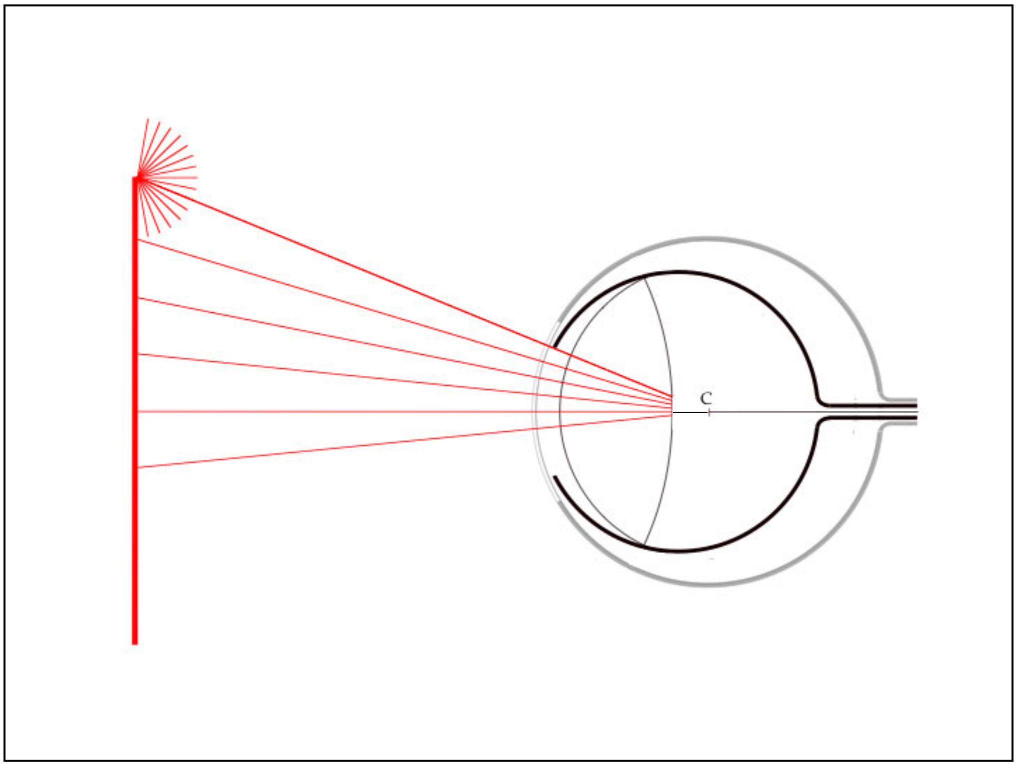
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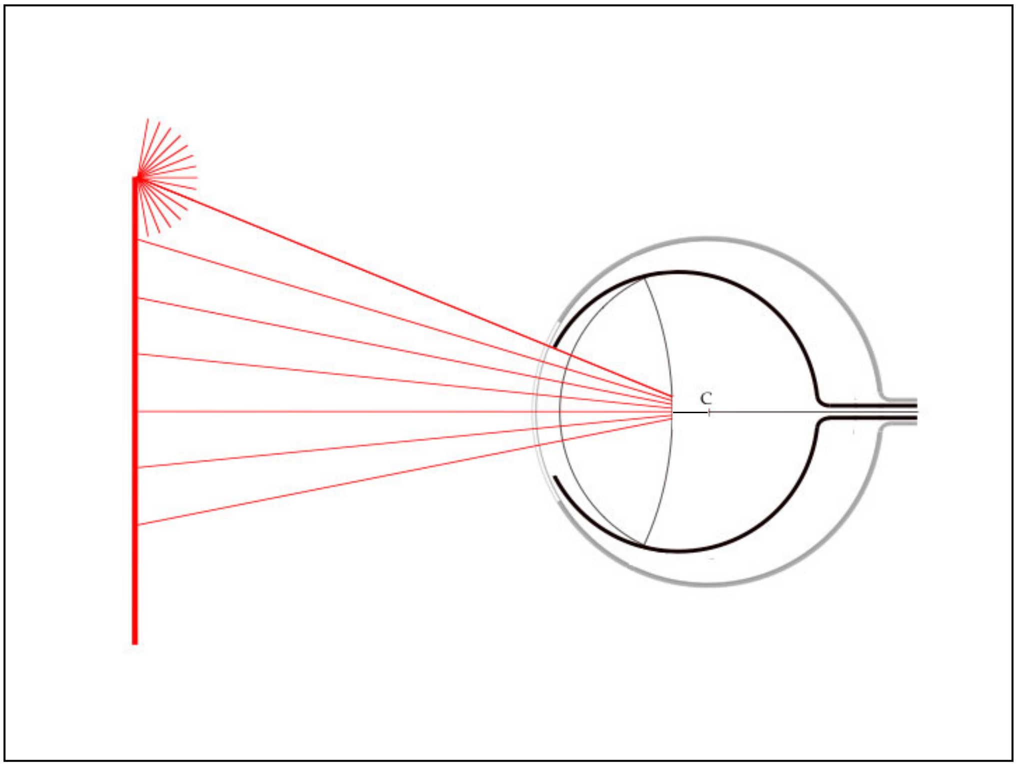
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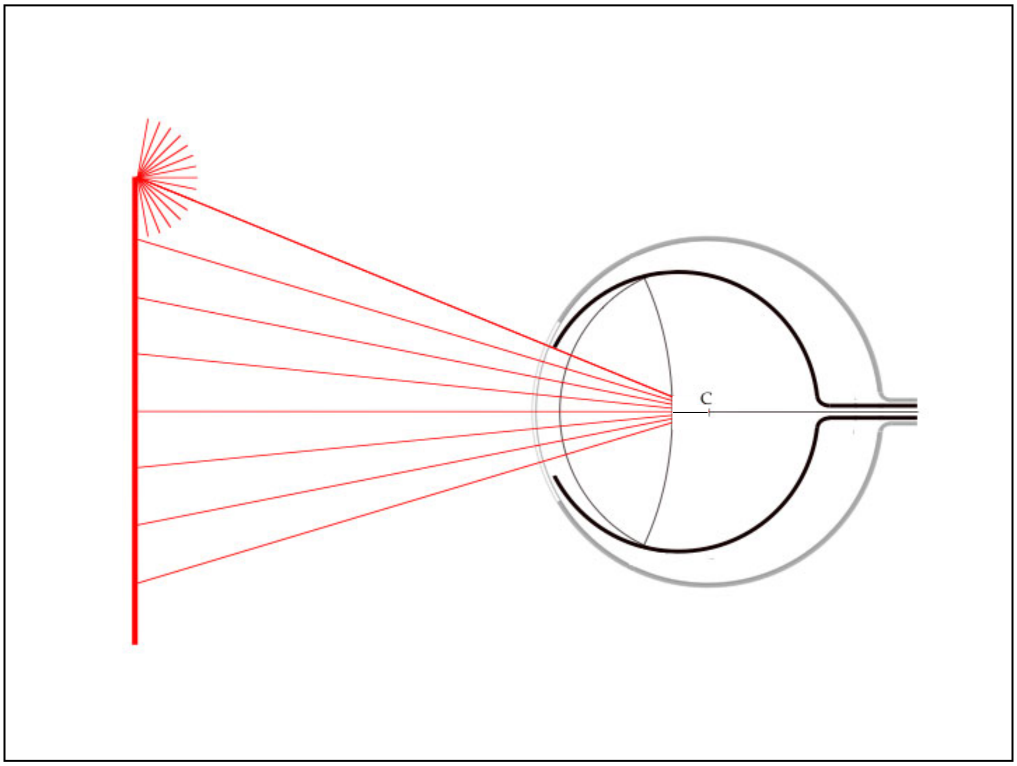
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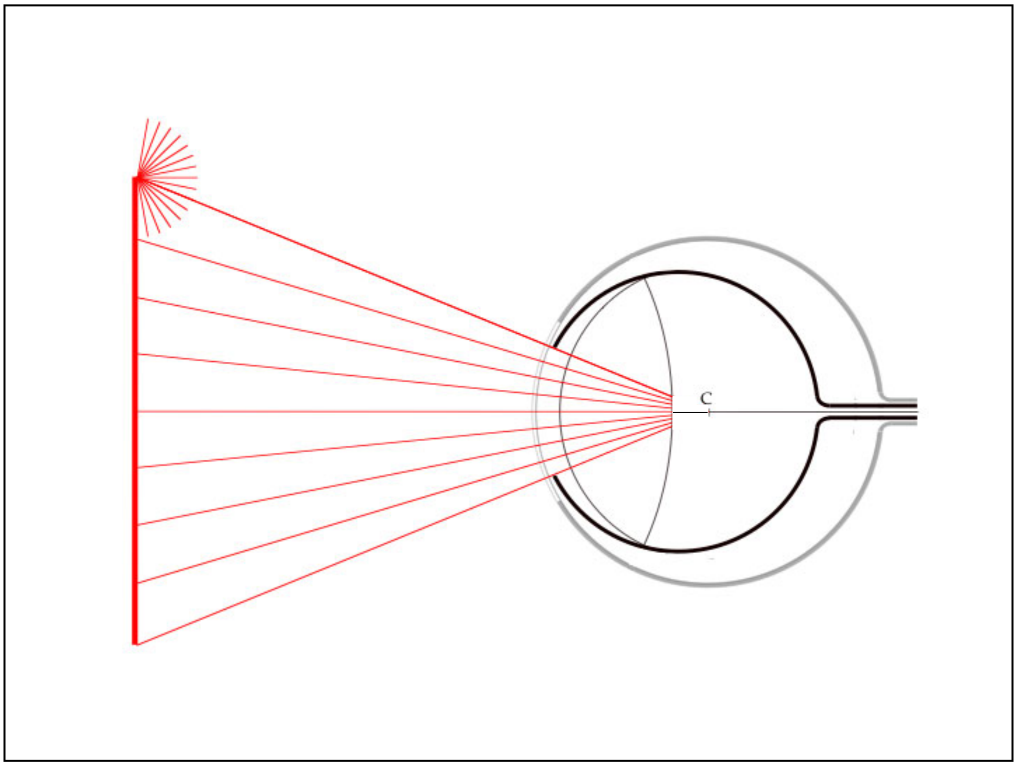
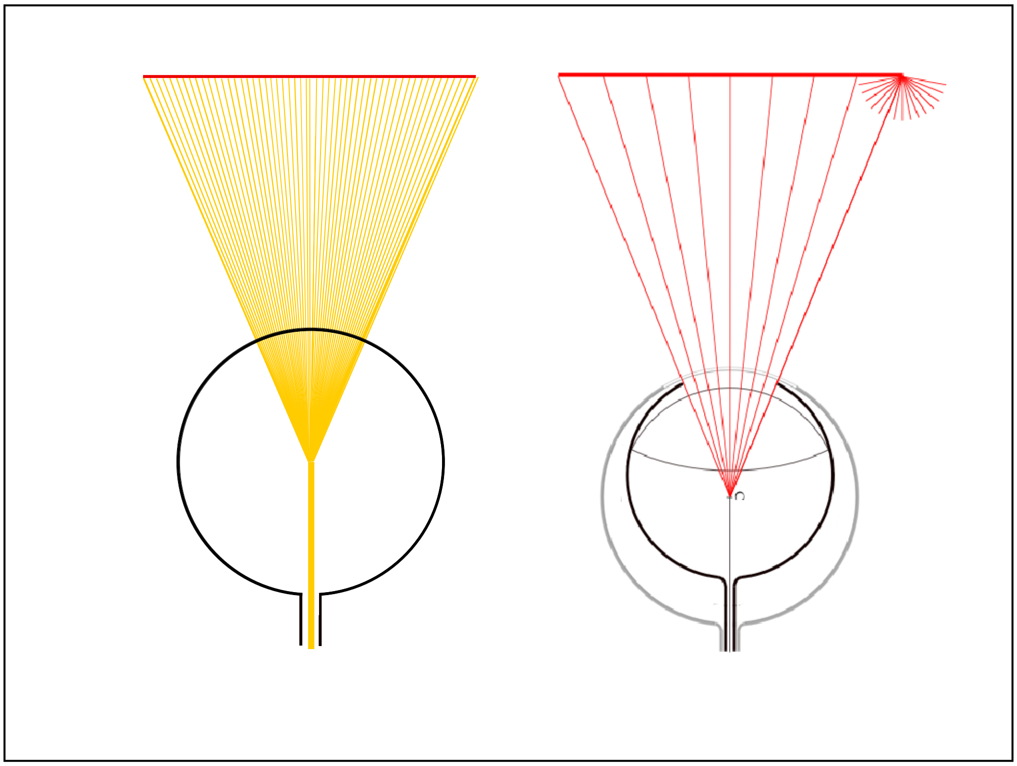
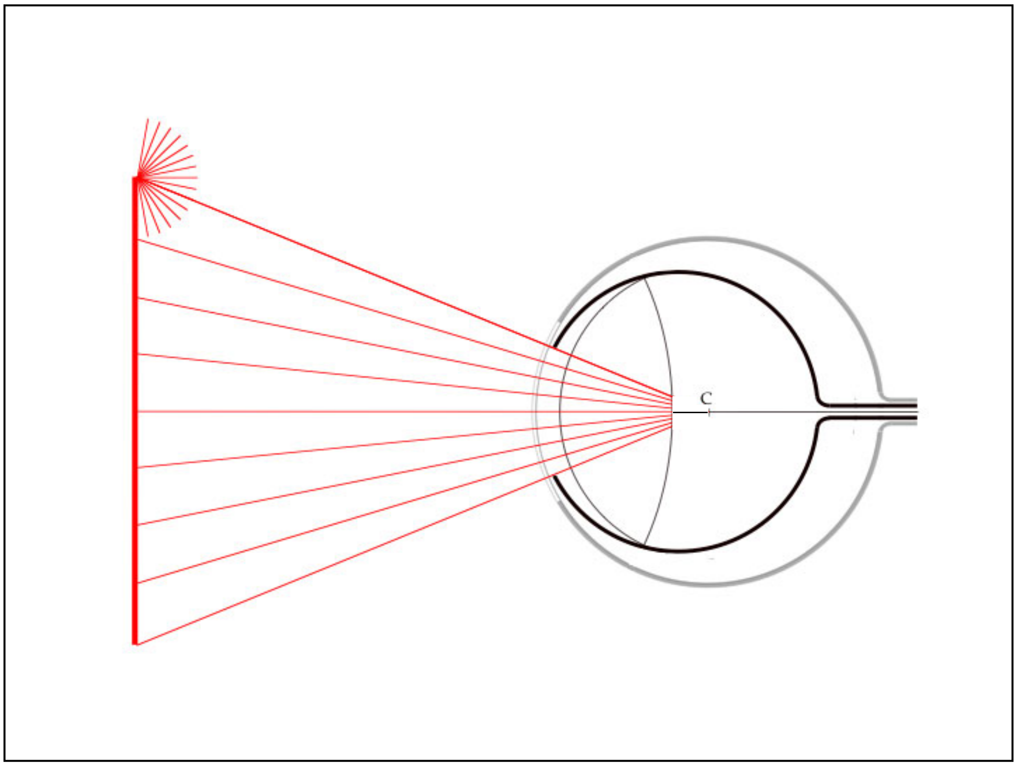


All of these rays, along with the infinitude of those between them, form a cone of light-radiation with its vertex at the center of the eye and its base in the field of view. As is evident from a side-by-side comparison,…

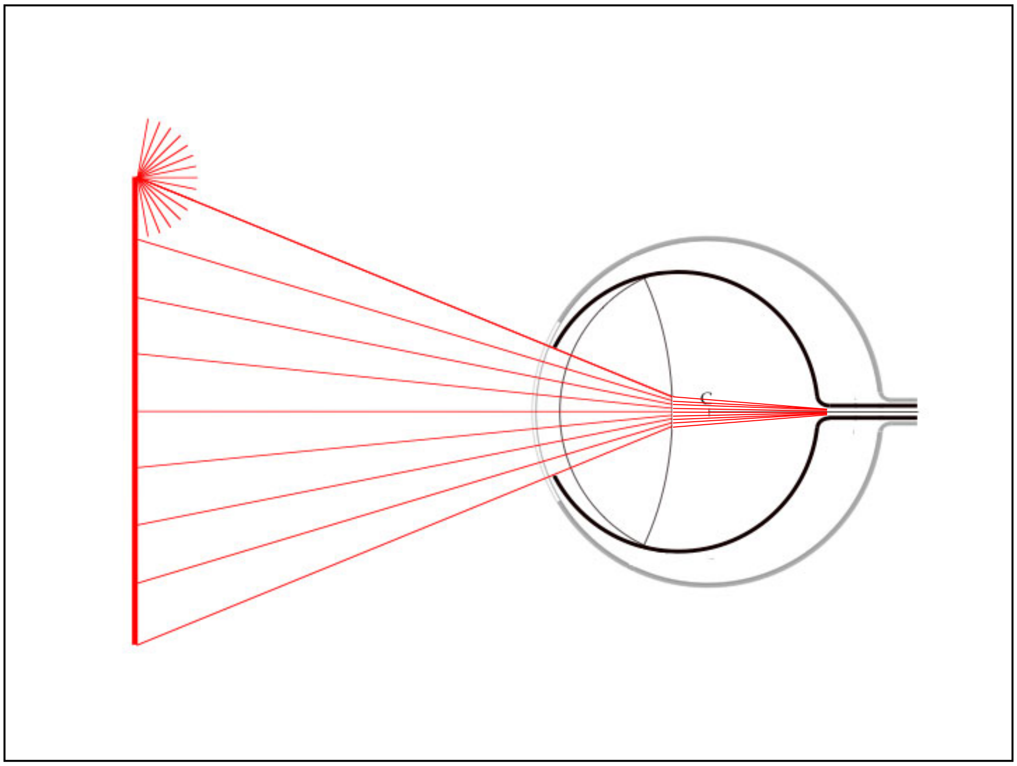


…Ibn al-Haytham's cone of light-radiation is geometrically equivalent to Ptolemy's cone of visual flux. The only difference lies in the direction of radiation: Ibn al-Haytham's is inward *toward* the eye, Ptolemy's outward *from* the eye. So here we have a textbook example of adaptation and assimilation on Ibn al-Haytham's part. We also have a clear indication of how deeply indebted he was to Ptolemy in forming his own account of vision.

Every ray within Ibn al-Haytham's cone of radiation makes a visual impression on the front surface of the lens, and all the resulting impressions pass as a whole and in proper order through the lens toward the center of the eye.

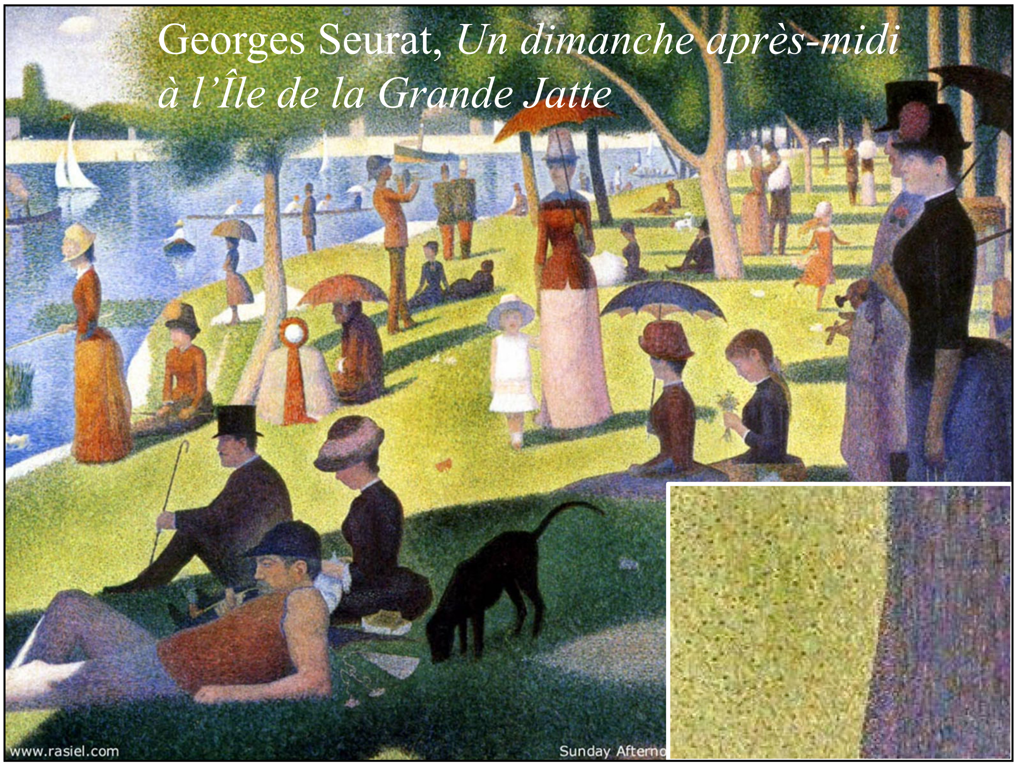


On reaching the rear surface of the lens, they're refracted into the vitreous humor and from there continue in proper order into the hollow optic nerve.



Still in proper order, they pass through the optic nerve to the brain, where they are subject to perceptual and intellectual processing.

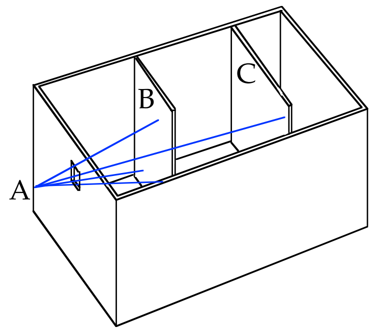
Altogether, the visual impressions made by the impinging rays on the lens's front surface form a sort of mosaic or pointillist representation of the objects in the field of view. That representation can be thought of by analogy to this famous work…



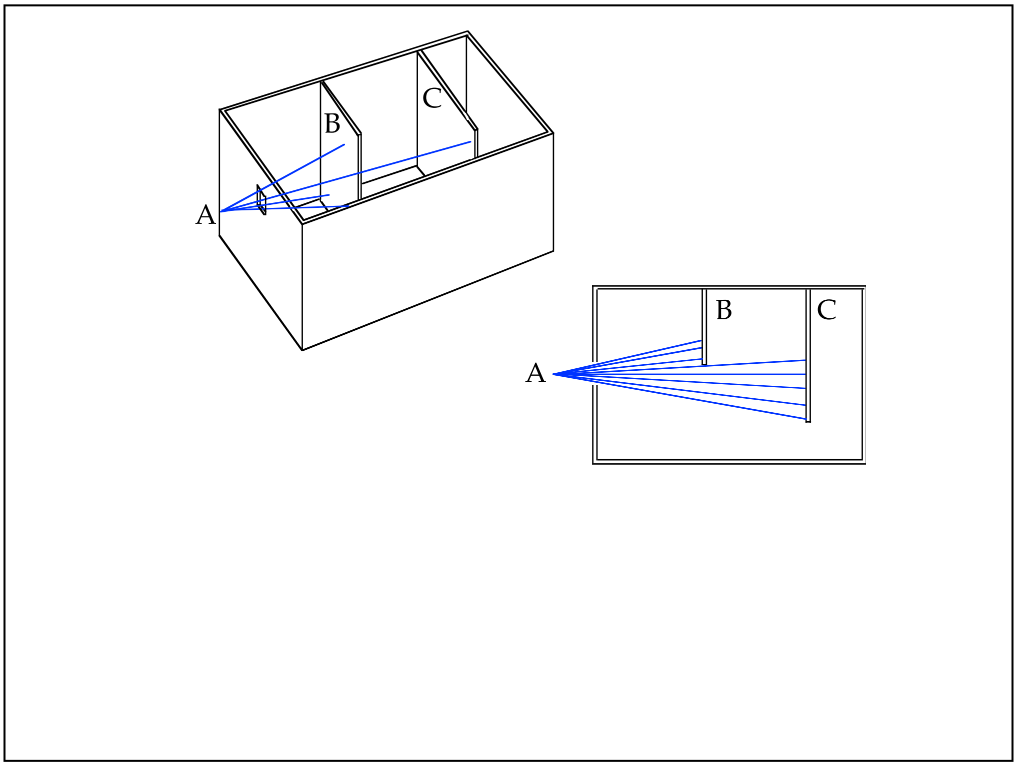
…by the French artist, Georges Seurat, who painted not with brush strokes but by making innumerable pinpoints of color with the tip of his brush—which is obvious from the enlarged inset at the lower right of the slide. Like this painting, the visual representation formed on the lens's surface consists of infinitesimal dots of color—visible rather than physical color—juxtaposed in a flat, two-dimensional array.

So how do we determine distance according to this account of sight? Clearly, we can't do so through an innate sense of ray-length because, instead of reaching out from us, the rays come to us from outside. It is as if we were being touched by someone else’s finger rather than using our own to establish contact. Consequently, just as we can’t determine by physical touch how far a projectile that hits us has traveled, we can’t determine by visual sensation how far the luminous color that strikes our lens has traveled. This means that we can’t sense distance intuitively or immediately, as Ptolemy’s theory would have it. Distance perception must involve an interpretive process of some sort.

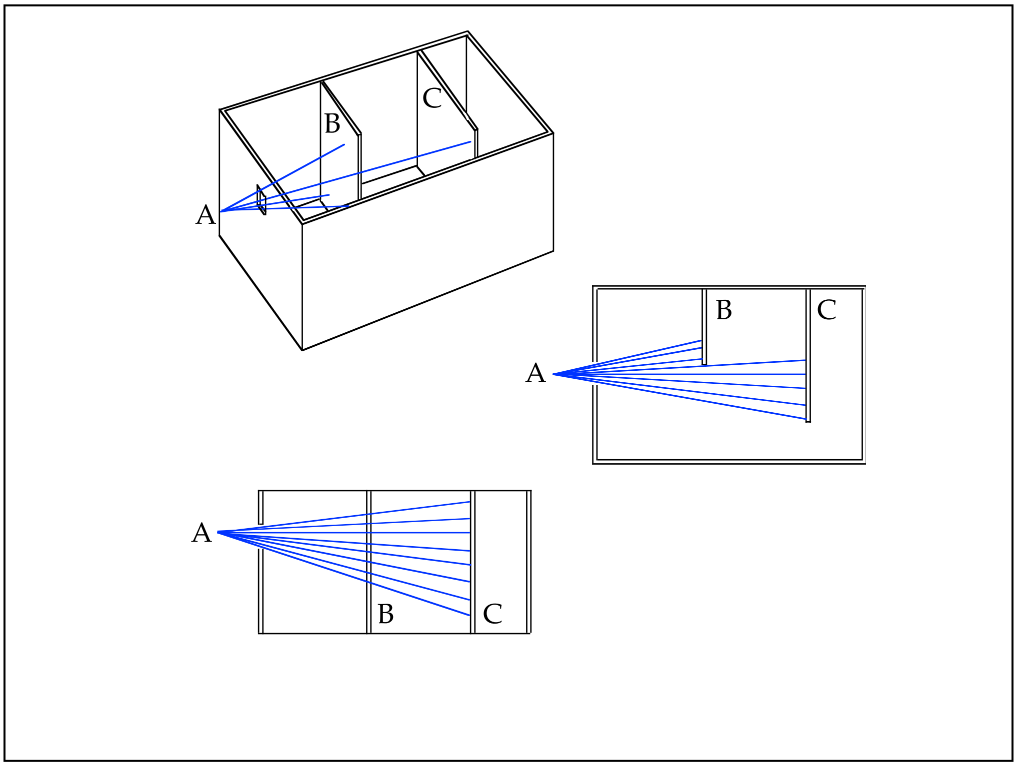
As empirical proof that we don’t perceive distance directly or immediately, Ibn al-Haytham offers the following experiment.



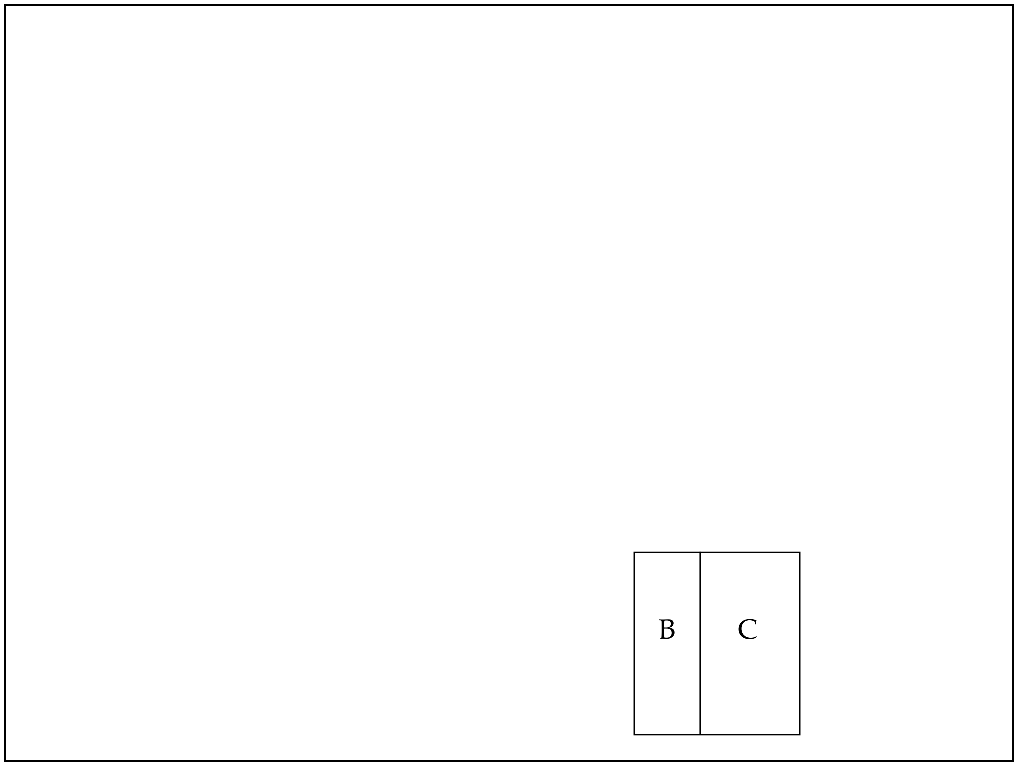
Let this be a room inside of which two walls, B and C, are set upright so that wall C lies behind wall B and juts out beyond it to the right. This arrangement is represented here,…



…when seen from above, and here…



…when seen from the side. Bore a small opening or window in the outer wall of the room facing the two inner walls, and place your eye at A so that you can see those two walls but not the room's sides, floor, or ceiling. In other words, assume that only the rays represented by blue lines, and all the ones between them, can reach your eye through the window. When you view the two walls under these conditions, as represented here,…



…they will appear to lie at the same distance and in the same plane but contiguous to one another. In other words, wall C won't appear to lie behind wall B but right next to it.

Ibn al-Haytham's point is that, far from being intuitive and immediate, distance-perception depends on context. Or, to put it another way, we infer distance from clues provided by the context within which we view things. Here's an example.

Within this restricted framework, it is impossible to tell how these two upright poles are situated with respect to one another. In fact, they both appear to lie next to each other in the same plane. But in proper context,…

…their spatial relation becomes clear according to various clues in the landscape—the way the poles are aligned with one another, the way they decrease in size from right to left, the way the scuff marks in the dust give the landscape texture, and the way the low hills in the distance provide a sense of spatial scope. On the other hand, in a virtually featureless landscape such as this…

…it's difficult, if not impossible, to gauge distances with any accuracy, especially as we look farther out toward the horizon.

With all this in mind, let's return to our pointillist painting.

As we noted earlier, it is analogous to the two-dimensional visual image produced on our lens. Therefore, it contains no depth. Yet from the various contextual clues provided in this two-dimensional representation, distances within it are fairly easy to determine. For instance, I'd estimate this distance…

…to be around 5 meters; this one…

…around 10; this one…

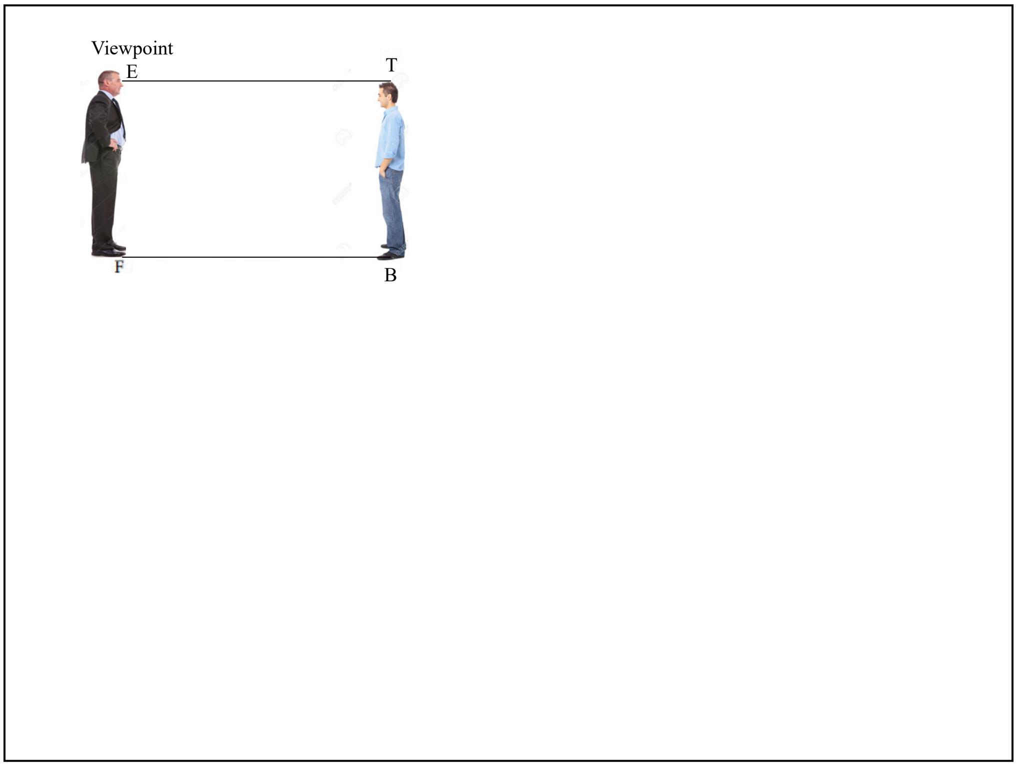
…around 25 or 30; and this one…

…close to 100. You may disagree with my numbers, but you'll all agree that they're in proper *relative* order and proportion. How, then, is it that we not only see but actually estimate distances in a non-spatial representation like this?

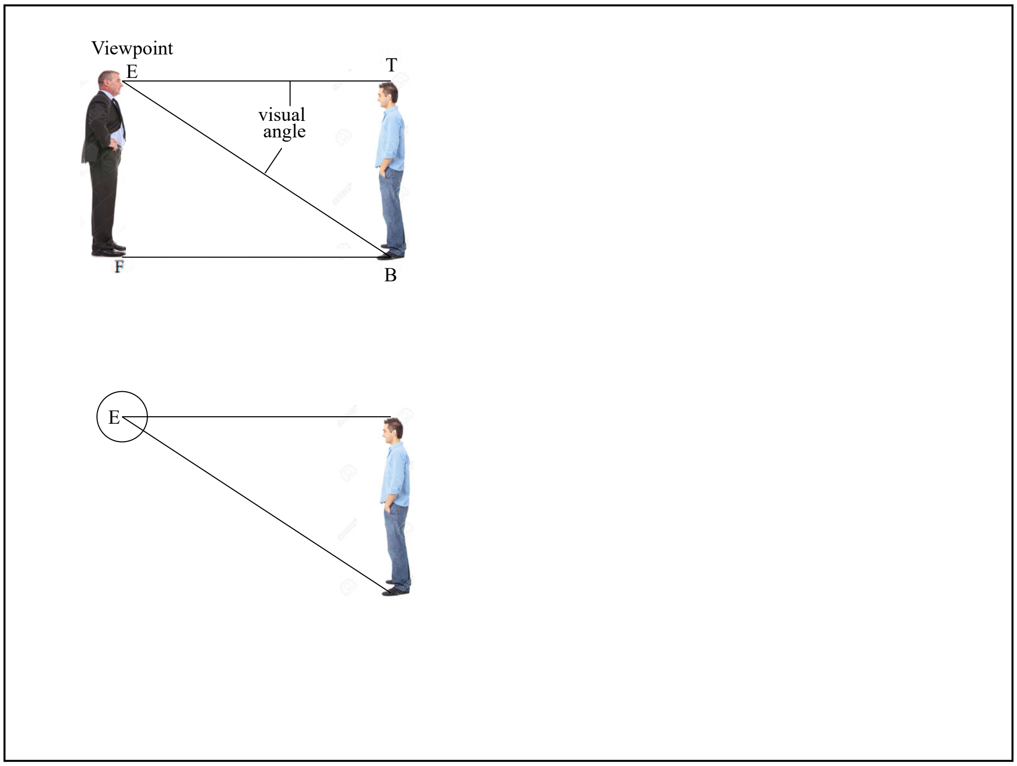
Ibn al-Haytham's response is as follows. When we first view the world as infants, all we see is a confusion of disparate colors without even recognizing what kinds of colors they are. Over time, though, repeated experience teaches us to distinguish among not only colors, but also objects. In that way, we develop a taxonomy according to which we distinguish red from blue or green, or horses from mules or oxen. This taxonomy is based on repeated perceptions of the same things or kinds of things, from which we form general, conceptual representations of them. These we store in memory as mental pictures of a sort. When we perceive new things, we compare them to these mental pictures in order to determine where they might fit in our taxonomy.

As we gain mobility, Ibn al-Haytham continues, we begin to measure the outside world and the things in it according to bodily units, such as arm-spans, arms-lengths, or paces. With repeated perceptions of such things as humans, for instance, we apply these measures to them in order to determine their size, concluding that adult humans are on average roughly one arm-span tall. We then include that determination in our mental picture. With increased mobility, we begin to extend these measures to several arms-lengths or paces away until we're able to judge reasonably long distances with fair accuracy on the basis of our mental pictures of those shorter ones. For longer distances we combine these known distance-measures, but in order to do so effectively, we need markers on the way. In the painting on the screen, these markers consist of the people and trees.

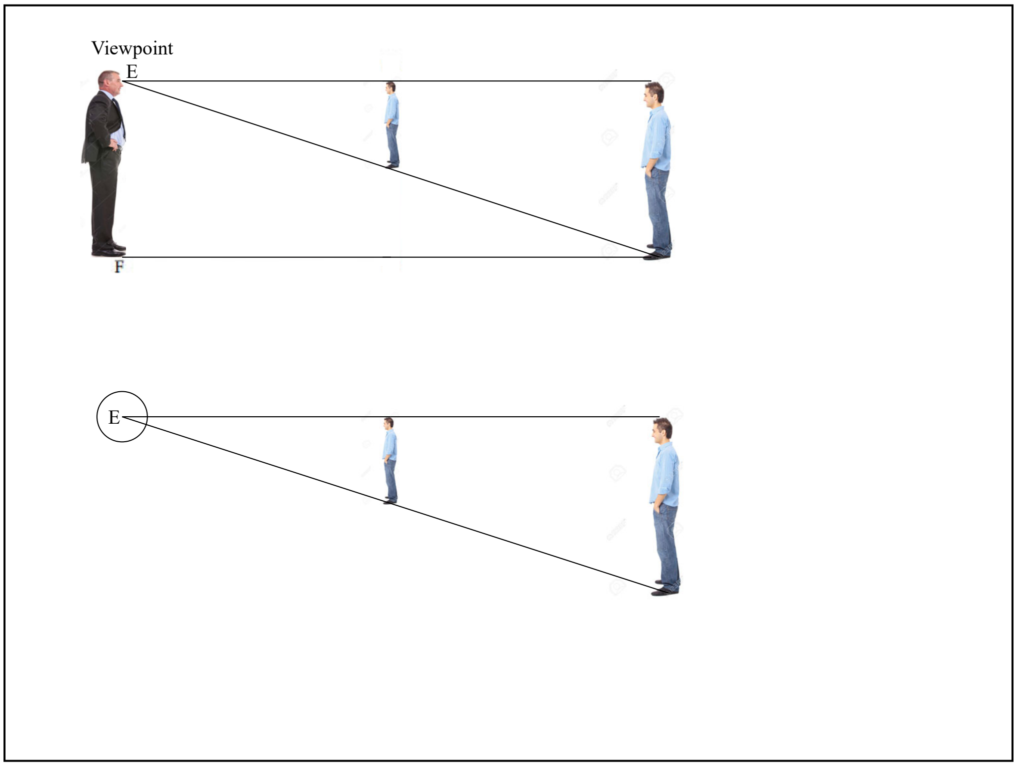
They *become* markers by virtue of what modern psychologists call the size-distance invariance hypothesis, which Ibn al-Haytham subscribed to at least implicitly. This is best explained geometrically.



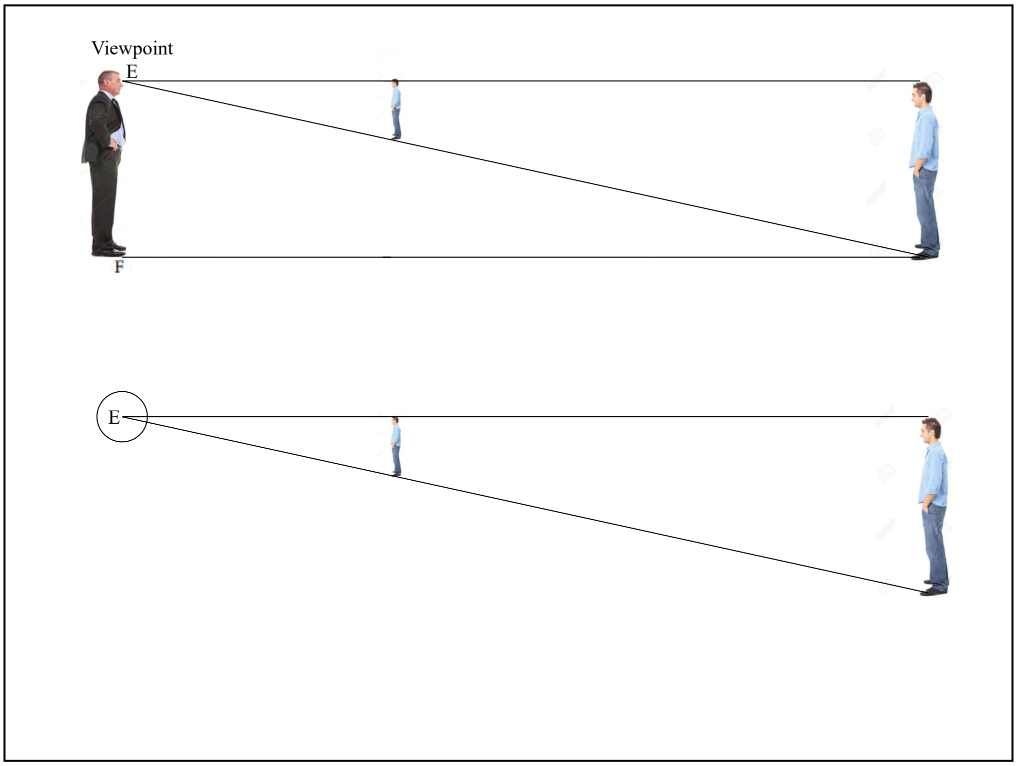
Let E in this diagram be the viewpoint from which, when standing upright with our feet at F to the left, we look at someone facing us. Assume that we know how far away he is, say three paces as measured by ET or FB. We've already determined that he's roughly one arm-span tall.



So when we see him, we see him according to the visual angle he subtends from point E at the center of the eye, where all the perpendicular rays coming from him converge This point Ibn al-Haytham refers to as the center of sight, and the angle formed there determines how much of the front surface of the eye, and thus of the lens, will be occupied by his visual representation. By correlating the relative size of that visual representation and its corresponding visual angle to the distance the person stands from the center of sight, we now know how large he will *appear* to be when seen from a distance of three paces. Having determined that, we now know how large *any* object one arm-span tall will appear to be from a distance of three paces, and we retain this knowledge in the form of the mental pictures mentioned earlier. Conversely, we know that any object one arm-span tall that *appears* to be this size lies three paces away.

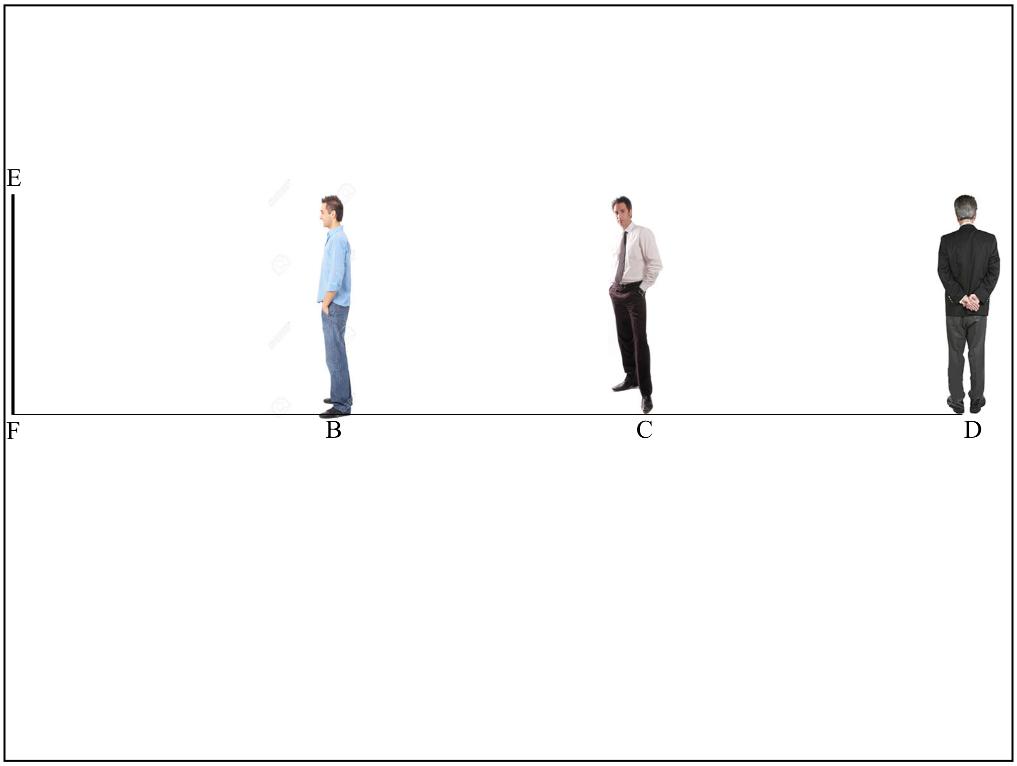


Now move the person twice as far away to a distance of six paces. He will be seen under a correspondingly smaller visual angle and will therefore appear to be correspondingly smaller. In fact, he will look nearly half the size he did at three paces away.

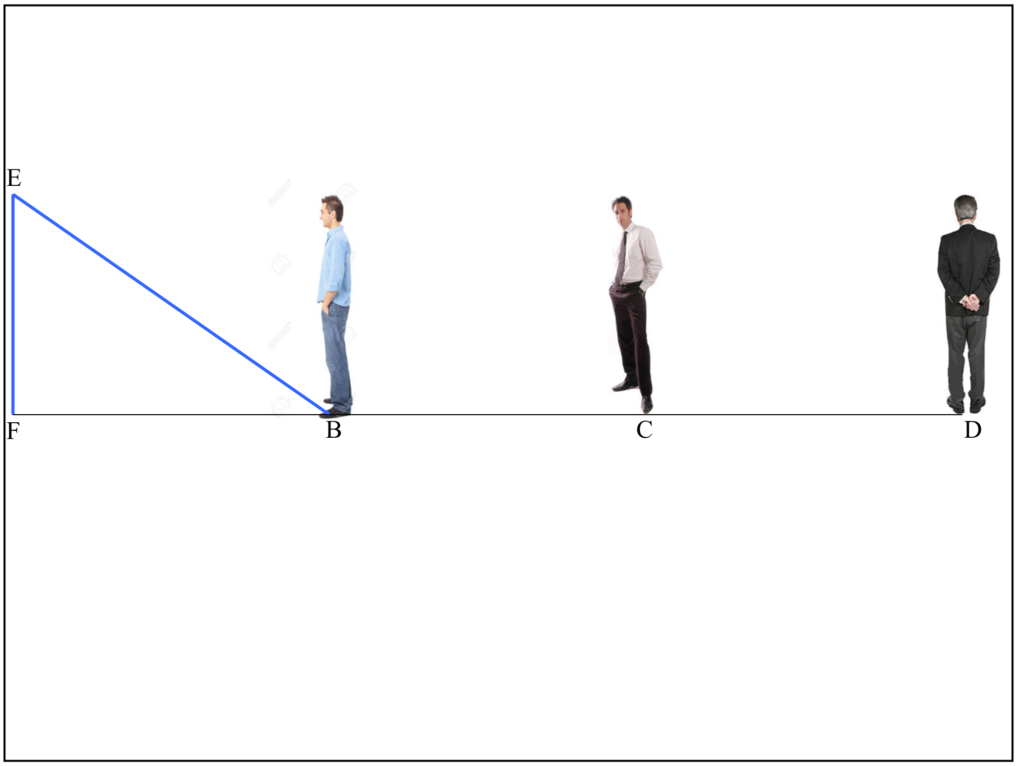


Move him another three paces away, and he will look correspondingly smaller, and so forth for every increment of 3 paces.

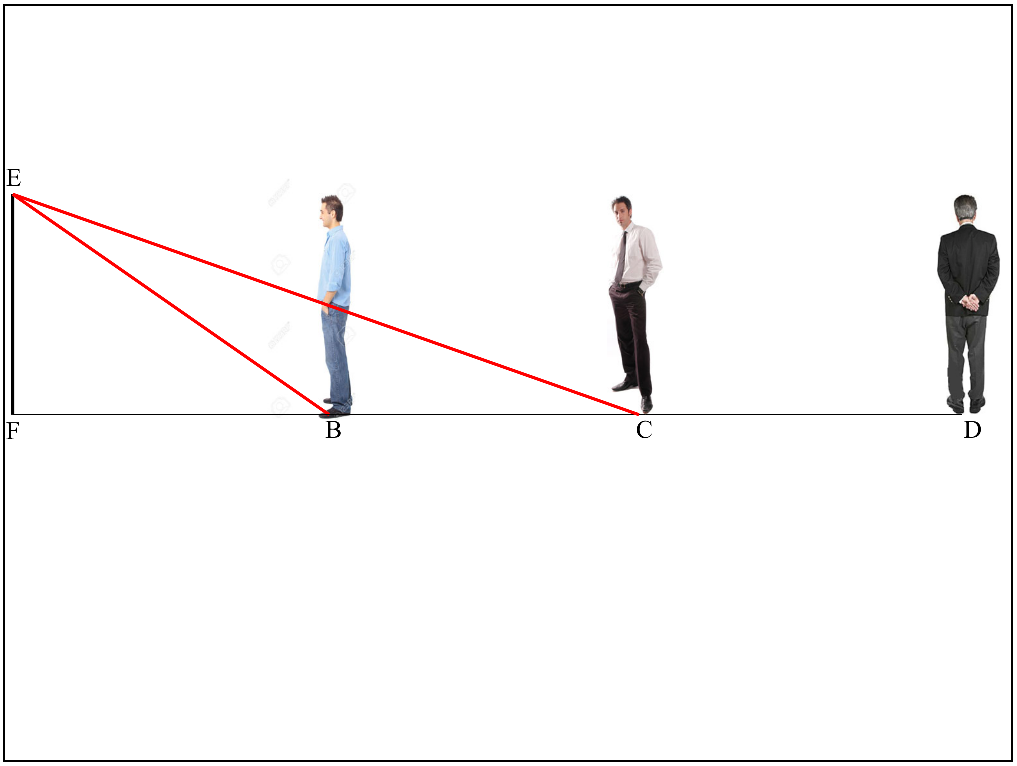
Continual perceptions of this sort eventually lead us to recognize how large many familiar objects of a known size should look from a wide variety of distances according to a wide variety of remembered visual angles. Not only that, but these perceptions also allow us to estimate expanses of ground according to the visual angles they subtend.



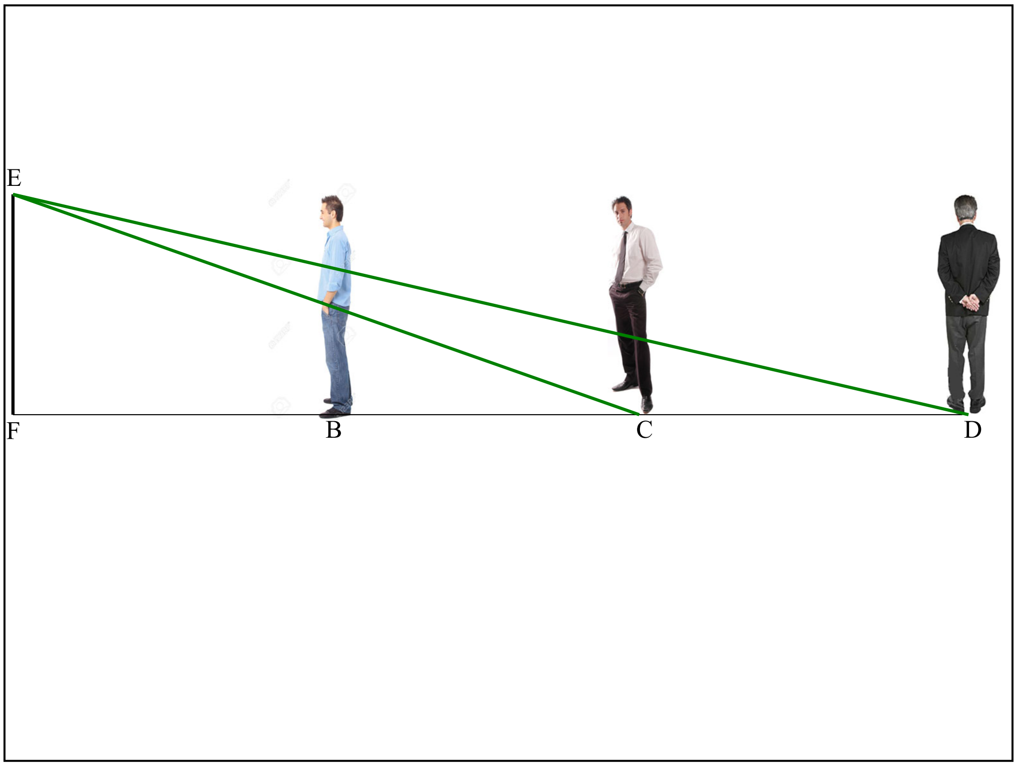
Having already determined that each of the expanses of ground FB, BC, and CD between the standing people here are three paces long, we know and remember that the expanse of ground at our feet that is three paces long will subtend angle FEB…



…formed by the blue lines. Consequently, when we see another expanse of ground at our feet that subtends the same visual angle, we know that it's three paces in extent. Likewise, we know from experience that an expanse of ground three paces long and three paces away will subtend visual angle BEC…



…formed by the red lines, whereas experience teaches us that an expanse of ground three paces long and six paces away will subtend visual angle CED…



…formed by the green lines.

Let’s return briefly to Seurat’s painting.

As you can see by the light blue line, several of the figures are at the same height as the viewpoint implicit in the painting. Let's concentrate on the two figures picked out by arrows: the woman in the middle and the couple behind and to the right of her. I'd estimate that from our implicit viewpoint the woman in the middle lies somewhere around 7 or 8 meters away. I'd also estimate that she looks around two-and-a-half times larger than the couple to her left,…

…according to their measure by the vertical blue line passing through them. I'd therefore judge them to lie around 35 meters away. I reached this judgment on the basis of at least three spatial clues implicit in the painting: the relative sizes of the two objects, their right-to-left separation, and the fact that the feet of the couple are above the woman's feet.

Here's the crucial point. Although the couple *look* around two-and-a-half times smaller than the woman, I nonetheless *perceive* them to be the same size.

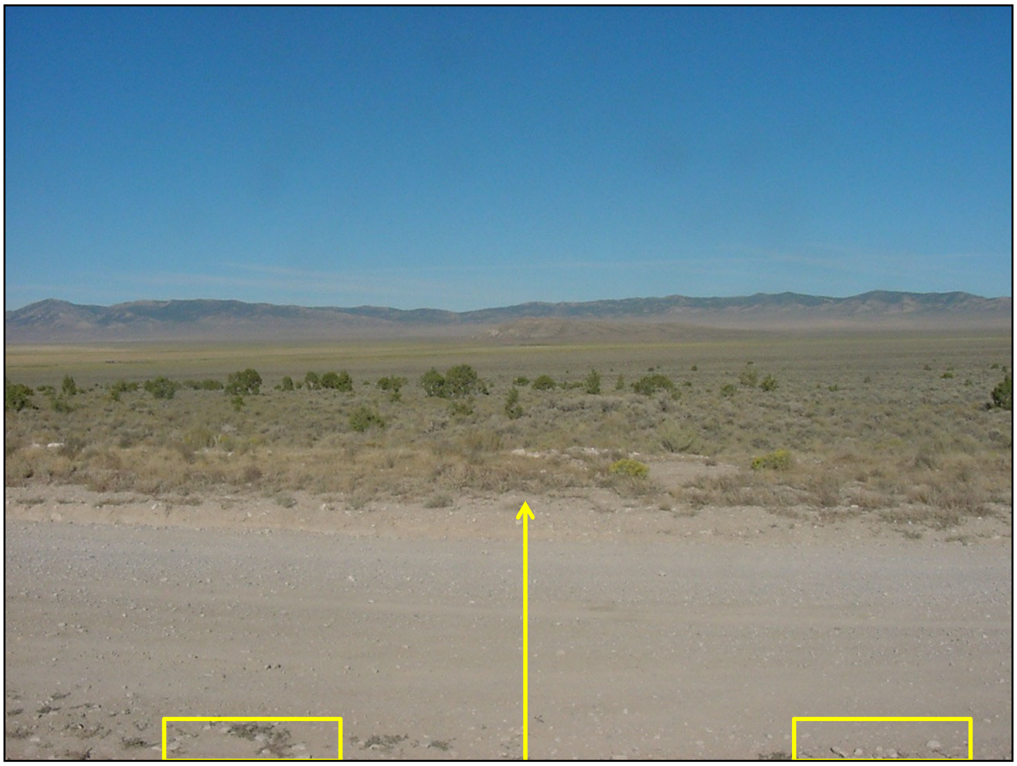
Here, too, I perceive this tiny figure to the woman's right to be the same size as her because I perceive his distance from both us and her to be quite large. According to Ibn al-Haytham, this perception of constant actual size is based on my correlating *apparent* size to estimated distance and comparing the result to remembered correlations. Furthermore, when I see unfamiliar objects of the same apparent size at the same estimated distance as familiar ones whose size I know, I perceive them to be the same a*ctual* size as those familiar objects. On that basis, I can determine the actual size of objects never seen before by comparing their immediate visual representation to my conceptual representation of commensurate familiar objects.

As Ibn al-Haytham explains it, then, size- and distance-perception entail deductive judgments based on correlating apparent size and estimated distance and then comparing the resulting correlation to known, memorized sizes and distances. Because we're so used to making such comparisons and judgments, Ibn al-Haytham explains, we're no longer aware of the deductive process leading to them because we carry it out so swiftly and automatically. That's why most of us assume that spatial perception is immediate and intuitive.

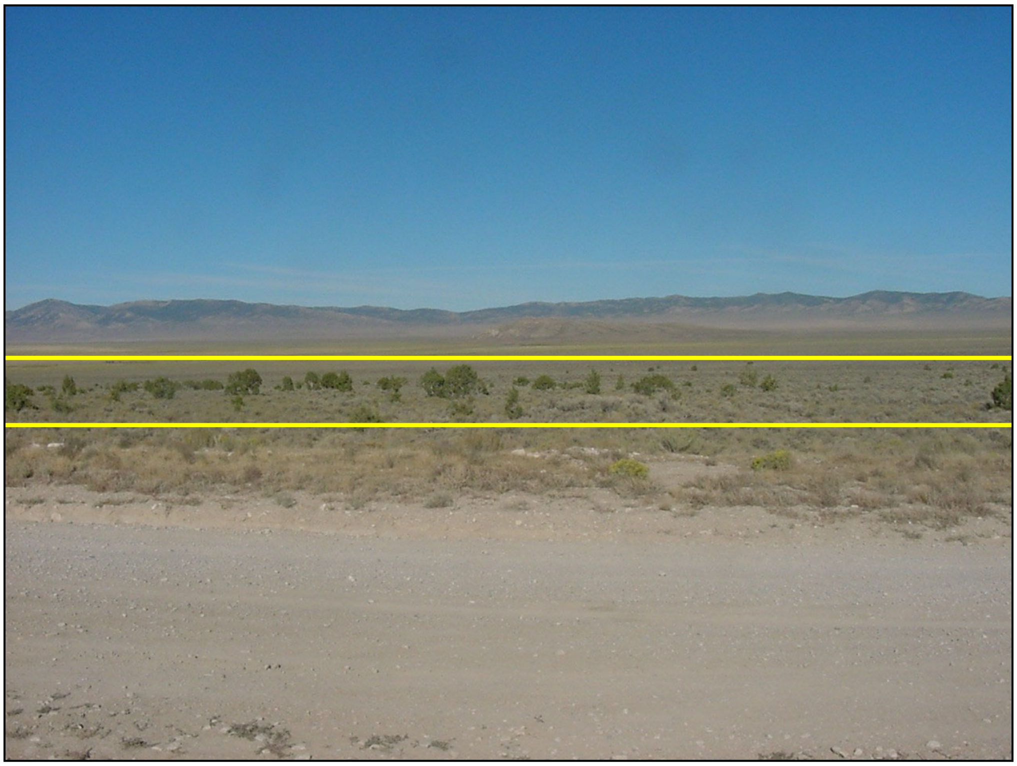
Here's one last example, which illustrates just how dependent size perception is on context and the ambient clues provided by it.

Take this landscape, which contains a wide variety of clues about distance and size. For instance,…

…judging by the size of these stones in the immediate foreground, the largest of which I estimate to be roughly 10 centimeters in cross-section, I'd estimate this distance,…



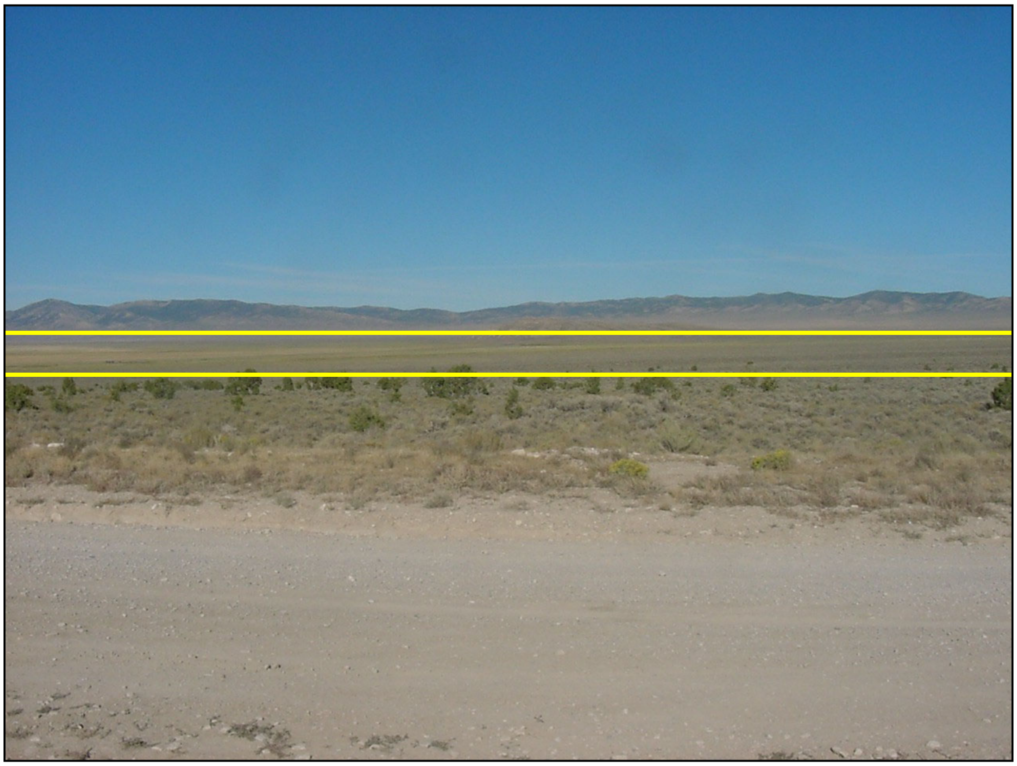
…from the immediate foreground to the far edge of the road, to be around 5 meters. Meanwhile, knowing that these…



…are piñon trees, which are typical of the high New Mexican and Colorado deserts, I’d estimate this tree…

…to be somewhere around two meters tall. On that basis, I'd guess this distance…

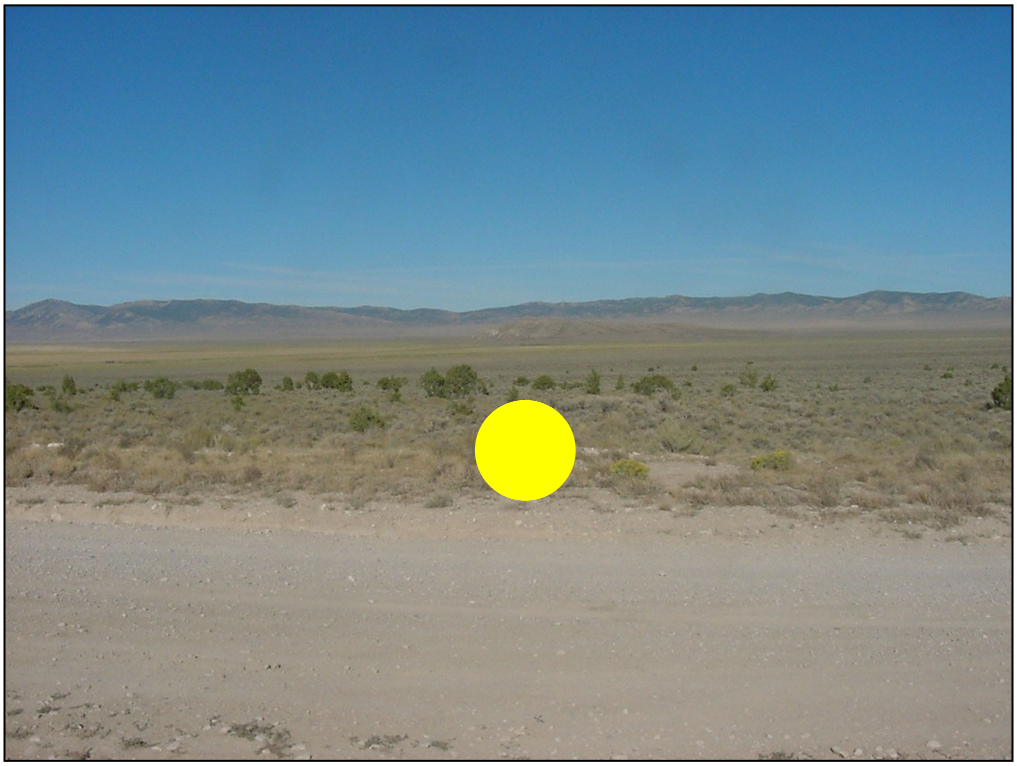
…to be around 75 or 80 meters. Finally, with those clues in mind and with a knowledge of the terrain pictured here, I'd estimate the low mountains in the distance to be somewhere around 5 kilometers away, although the lack of markers within this…



…range of the landscape makes that estimate extremely tentative.

Now, with all these points established, let’s assume that the yellow circle…

…here represents a moderately small object placed near us, in the immediate foreground of the picture. It looks to be maybe 30 or 40 centimeters in diameter in comparison to the largest stone near it. Let us move it away, just beyond the edge of the road at a distance of roughly 5 meters.



Even though it is precisely the same size and subtends precisely the same visual angle as before, it *looks* much larger because of the increased distance at which we imagine we see it. Finally,…

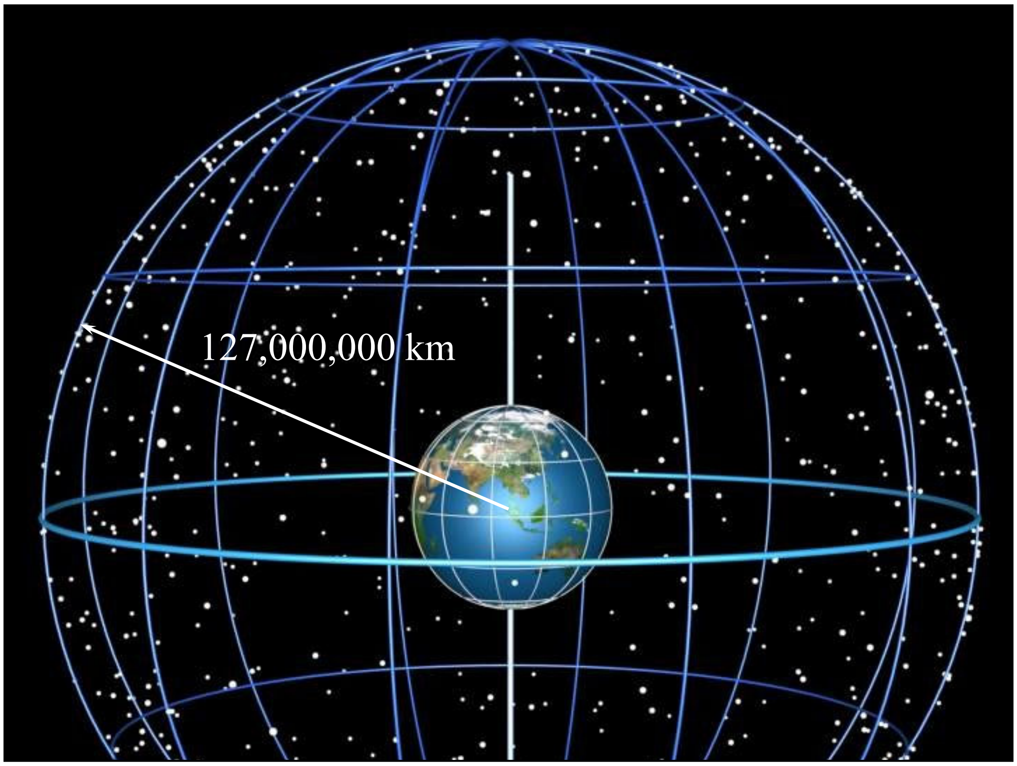
…when we shift it out to a point just above and beyond the mountains several kilometers away, it appears enormous, even though its actual size remains unchanged.

Armed now with what I hope is a fairly clear understanding of Ibn al-Haytham's account of distance- and size-perception, we're ready to conclude with a look at his explanation of the Moon Illusion. Actually, Ibn al-Haytham offers two, complementary explanations. One, which traces back to Ptolemy and even earlier, attributes the enlargement of Moon and Sun at horizon to refraction through thick, humid vapors rising to a relatively short altitude from the earth's surface. When viewed through such humid vapors along the horizon, the two bodies appear magnified in a way similar to that in which objects viewed through water…

…appear enlarged by refraction. This was how the Moon Illusion was commonly understood for centuries after Ptolemy.

After a careful analysis based on the principles of refraction established by Ptolemy, however, Ibn al-Haytham realized that the refractive magnification caused by humid vapors wouldn't be nearly enough to account for the degree of enlargement that the Moon and Sun appear to undergo at horizon. There had to be an additional, more fundamental cause, and it had to be based on something other than the physics of refraction. Ibn al-Haytham was thus ultimately driven to seek that cause in the psychology of perception—or, rather misperception—and, more specifically, in the misperception of distance and size. So let's take a look.

Ibn al-Haytham opens his explanation with a couple of astronomical considerations. The first, which accords with the accepted cosmology of his day, is that the vault of the heavens is spherical with its concave surface facing us.



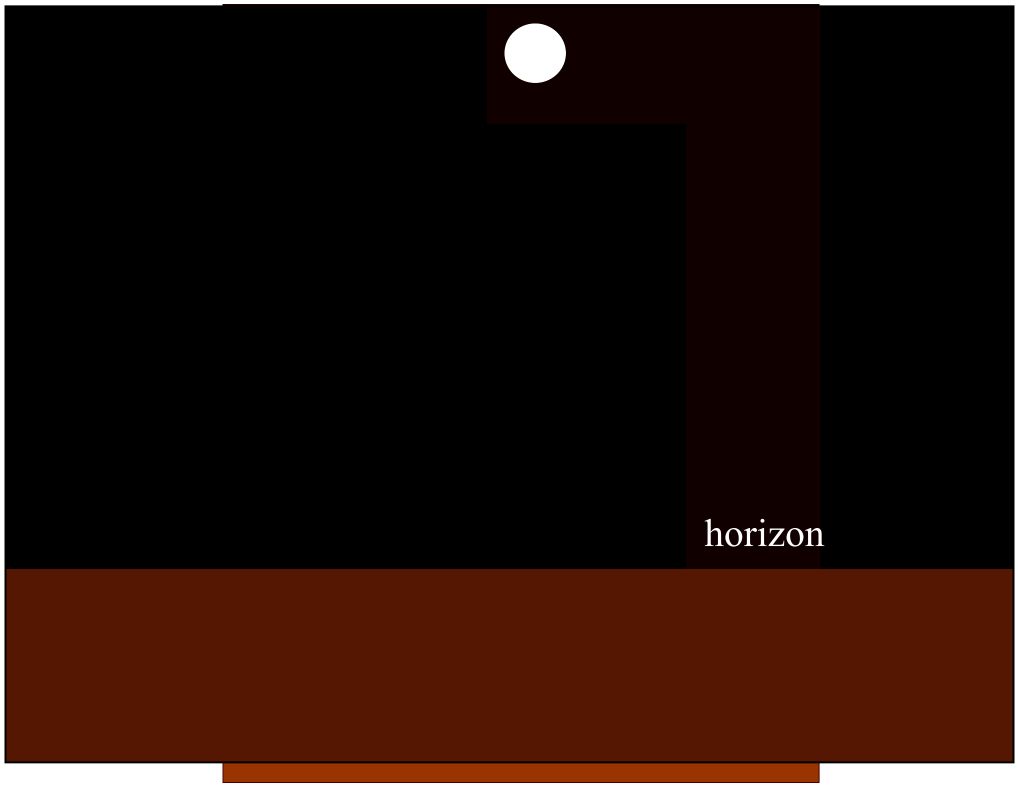
But that spherical vault is so far away—around 127,000,000 kilometers or 10,000 times the earth's diameter according to the calculations of his day—that we can't perceive its distance properly because we lack any markers to help us along the way: no trees, no expanses of ground, no mountains at the horizon to help us put its distance in perspective. As a result, we can't perceive its concavity or any other spatial feature of it. All we can perceive is its blue color during the day or its star-studded blackness at night. Still, we're constrained to guess at its shape, and our best guess, according to Ibn al-Haytham, is that it forms a sort of ceiling extending high overhead and roughly parallel to the plane of the ground we stand on. This misperception is reinforced by cloud formations, such as this one,…

…that seem to follow the flat contour of the sky.

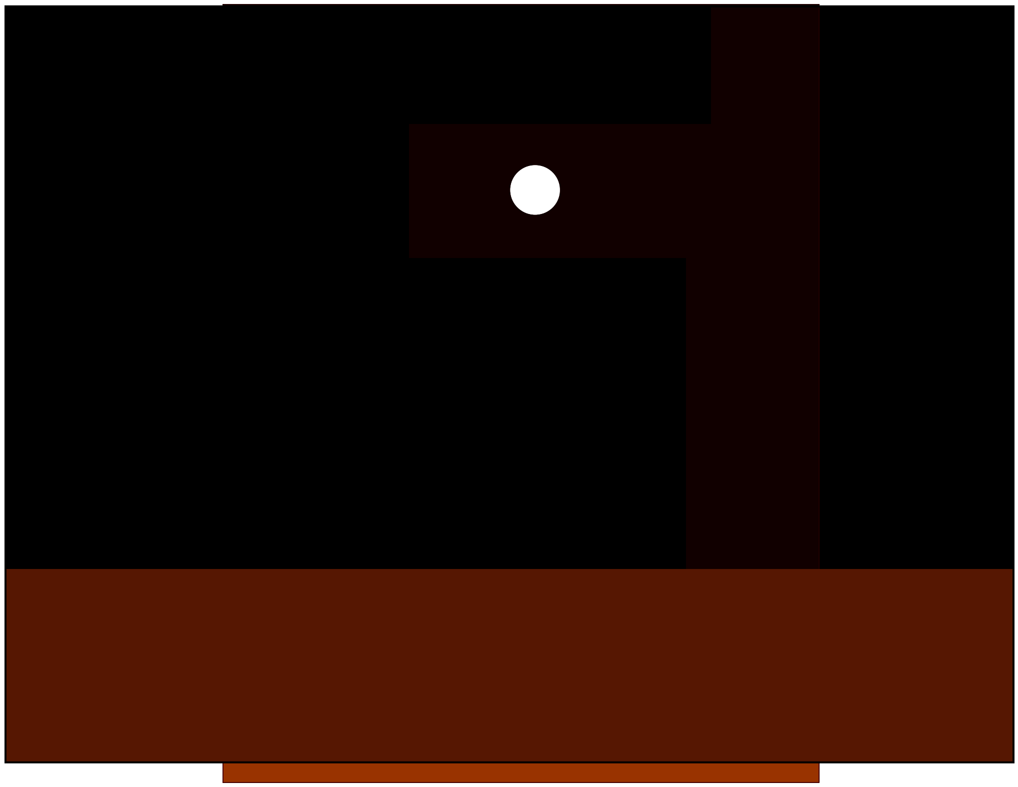
The second astronomical consideration is that the Moon and Sun move in circular orbits around the earth and concentric with it, completing each orbit in a 24-hour day. Like the celestial vault, these orbits lie so far away that, lacking any markers along the way to define their distances, we cannot detect their curvature. We therefore perceive them as straight lines along the heavenly ceiling. Consequently, both celestial bodies appear to move away from us in straight lines through the heavens, and as they do, they appear to recede ever farther from us as they move from zenith, directly overhead, to horizon.

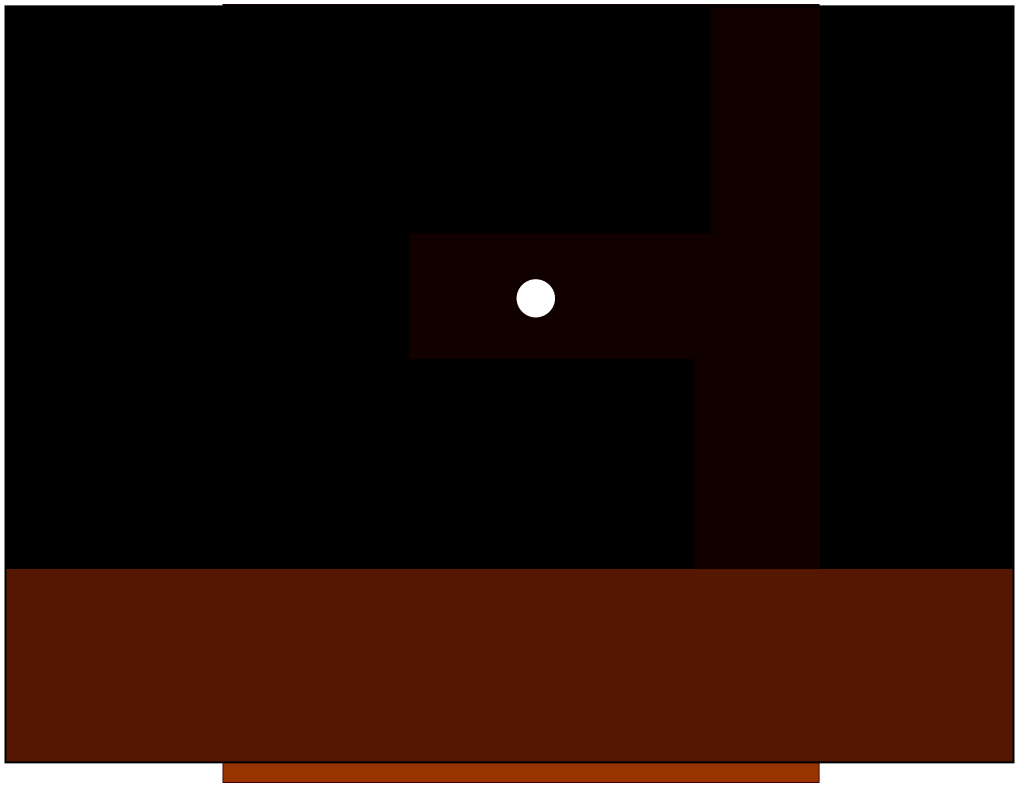
If they actually *did* recede from us along straight lines, though, they would appear to get smaller and smaller as they receded from us because they would subtend ever smaller visual angles. They would therefore appear to follow a pattern similar to this string of light-fixtures…

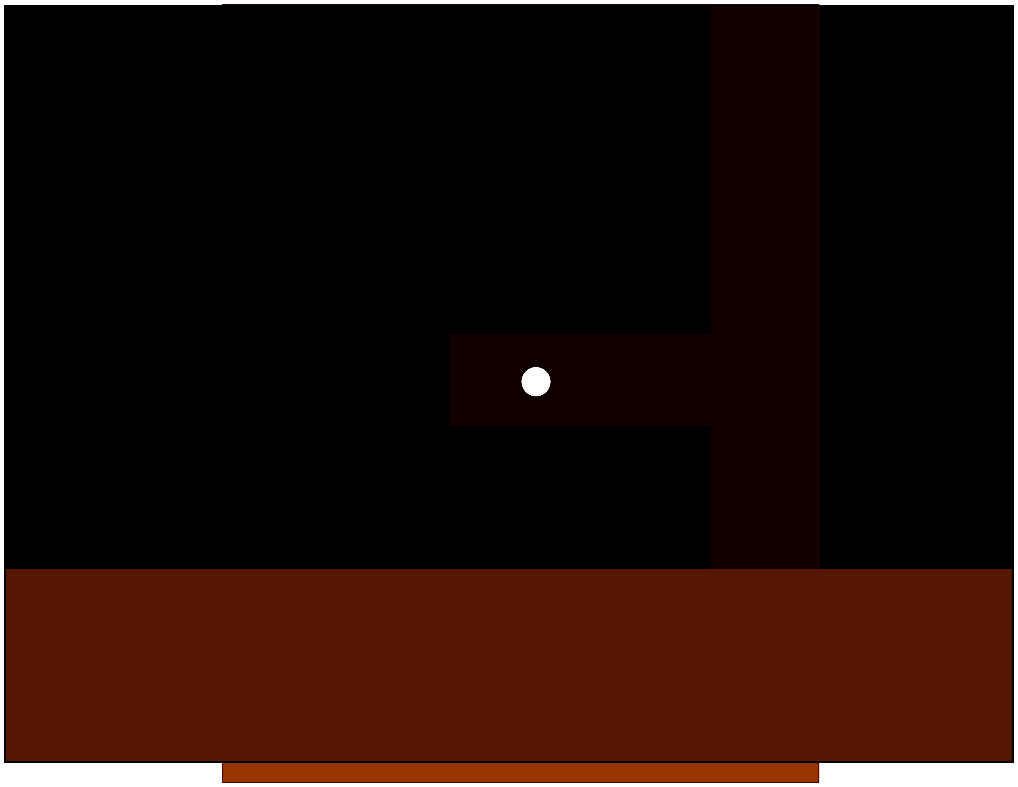
…on a ceiling viewed down a long corridor, continually decreasing in apparent size while marking out ever smaller distances as they approach the horizon. What we should see, then, is something like this,…

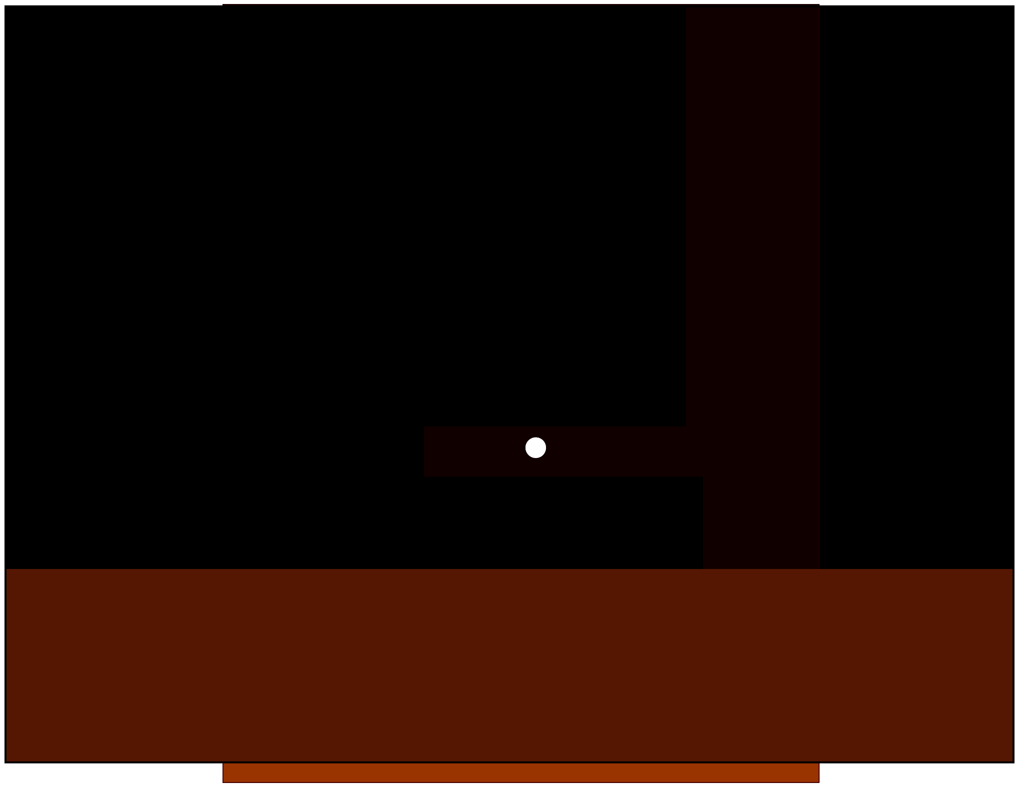


…starting with the moon high above, and then moving toward the horizon along continually decreasing distances in equal time increments, as follows…

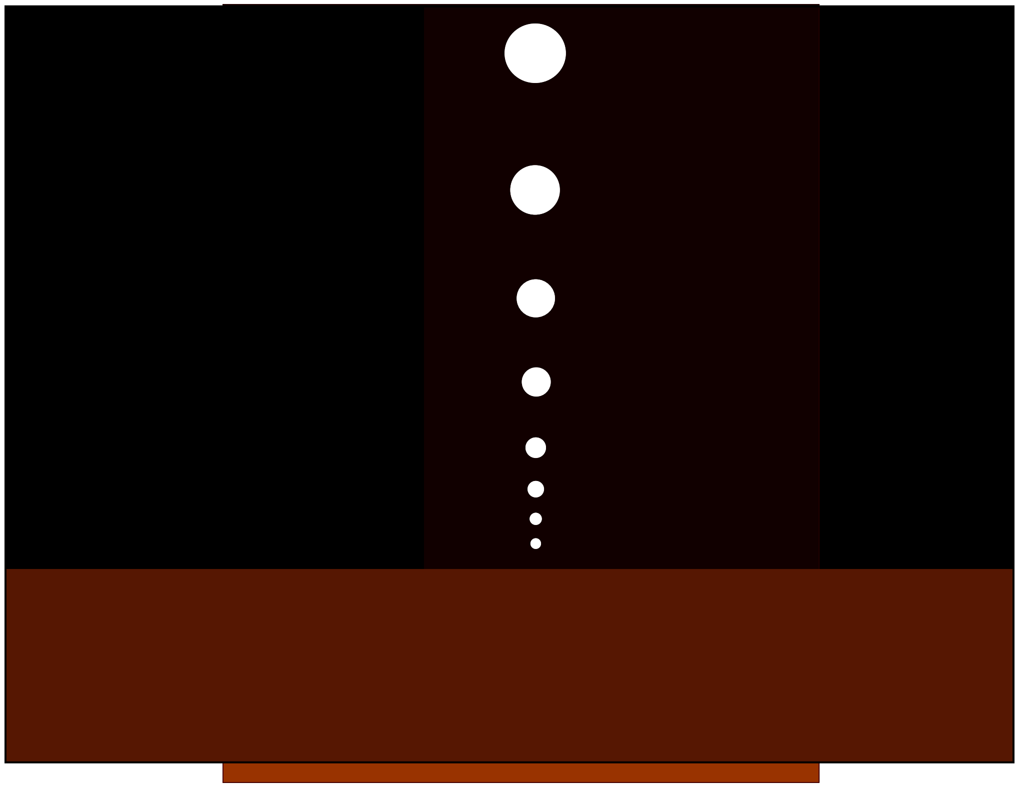








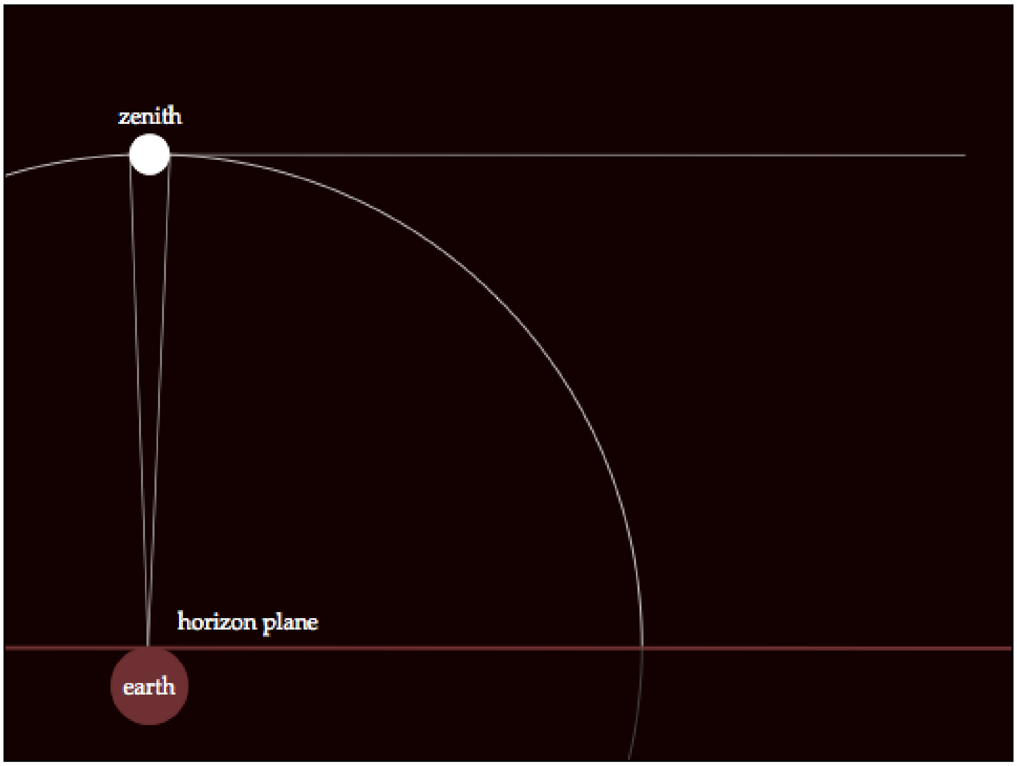
This,…



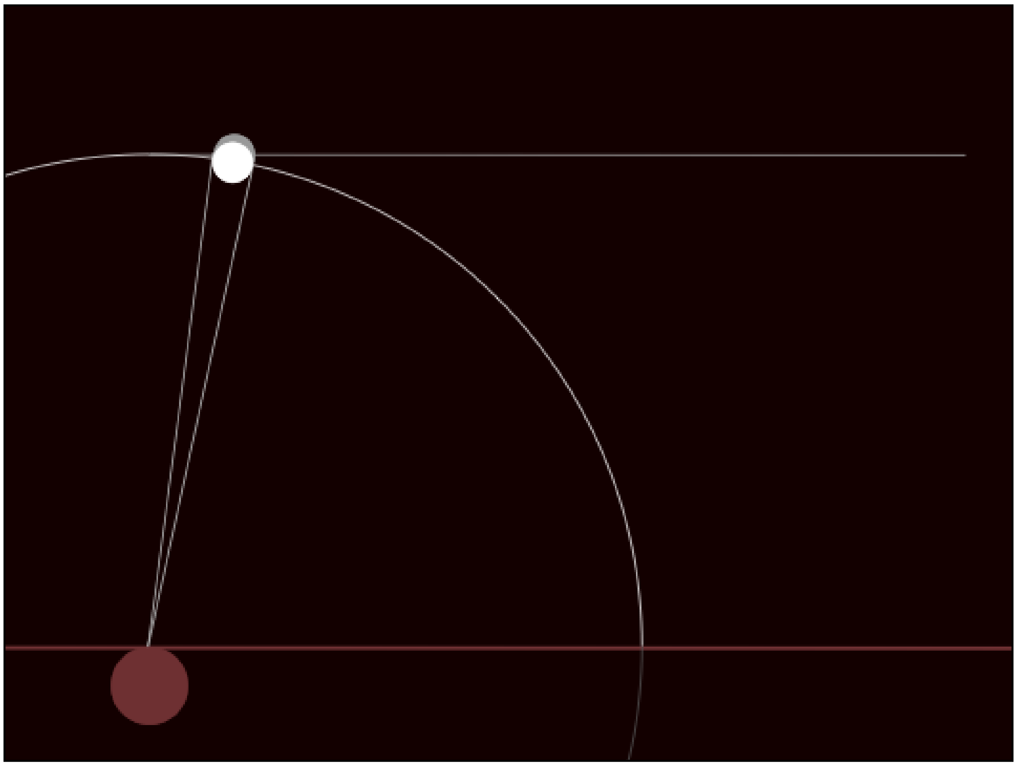
…then, is the final, overall pattern, which is just like the one we saw earlier with the light fixtures on the long corridor's ceiling.

But since the Moon and Sun actually revolve about us in concentric circles and therefore maintain the same distance from us throughout their orbits, they subtend the same visual angle throughout those orbits. Consequently, they look as though they grow in size as they approach the horizon because their *apparent* distance, which is perceived according to their apparent rectilinear motion away from us along the celestial ceiling, increases while their actual size, as measured by the visual angle, remains constant.

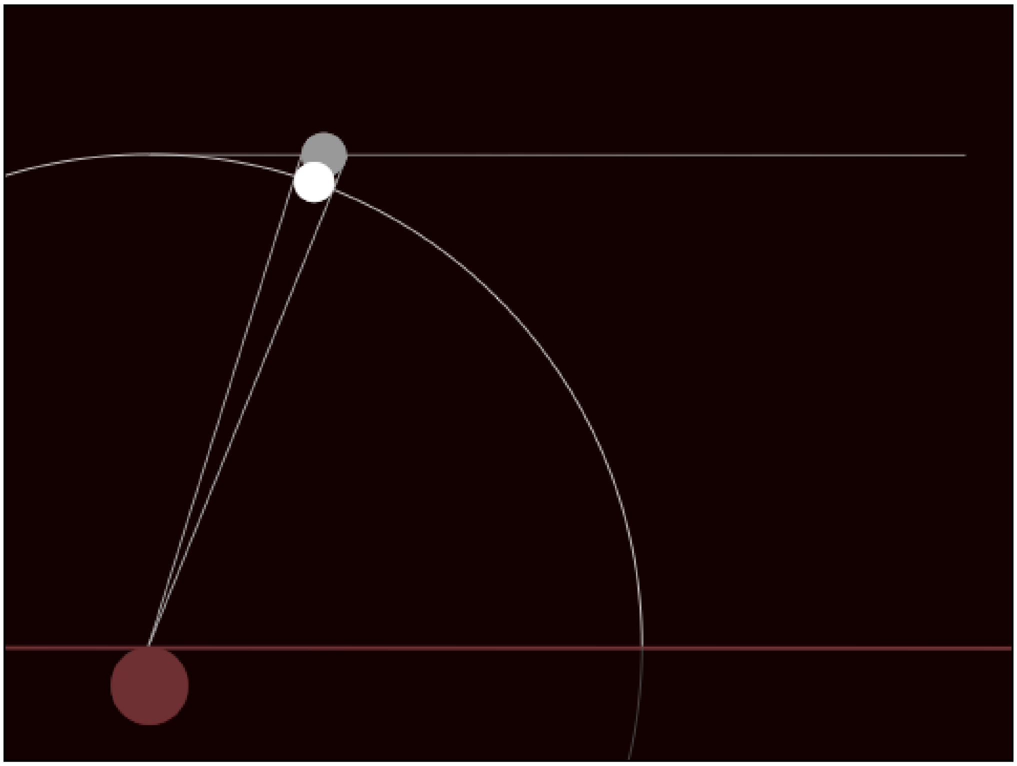
What Ibn al-Haytham is getting at is easy to grasp with the aid of a diagram.



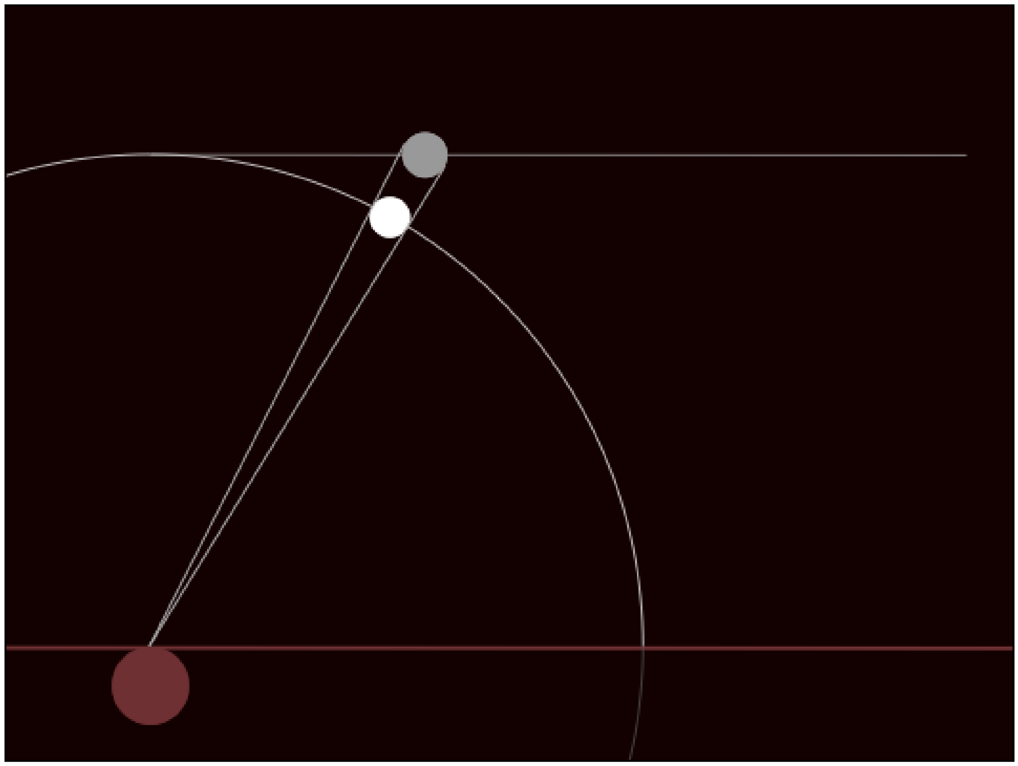
Let this represent the moon directly overhead at zenith, its actual orbit being the circular segment in white and its apparent path along the heavenly ceiling the straight, gray line tangent to it. The observer stands where the plane of his horizon touches the Earth's surface, and the visual angle under which he views the Moon is contained by the gray lines converging at his viewpoint.



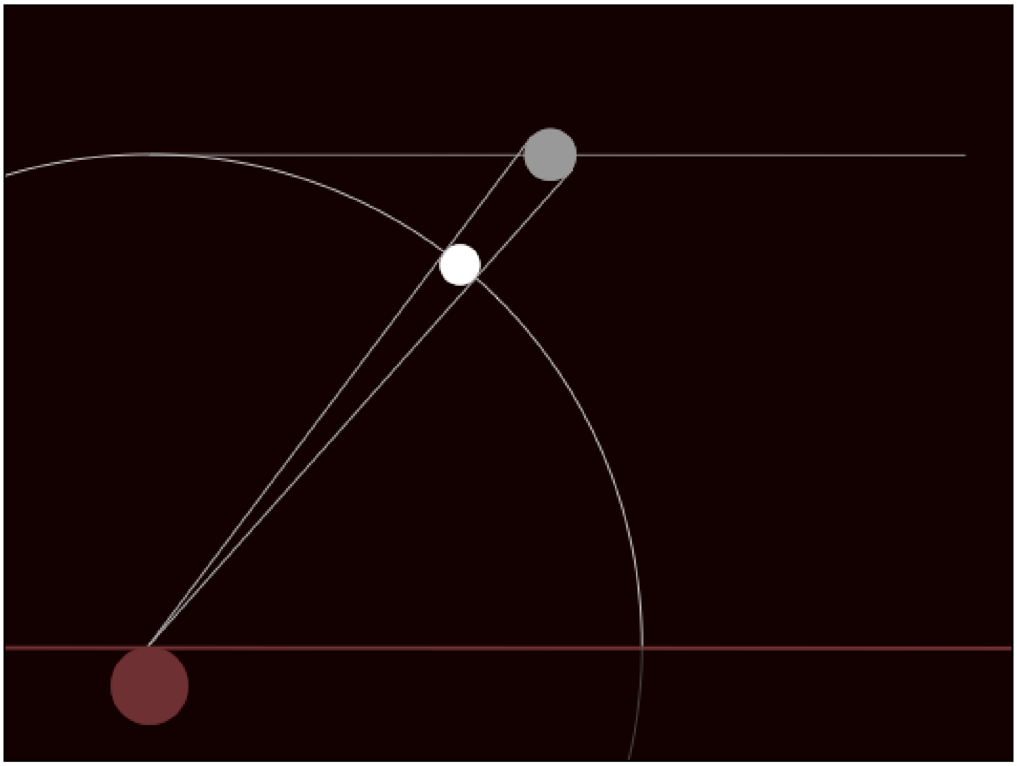
When the Moon passes through this part of its orbit, it will subtend the same visual angle, but the perceived Moon in grey on the rectilinear path will lie slightly above and ahead of it.



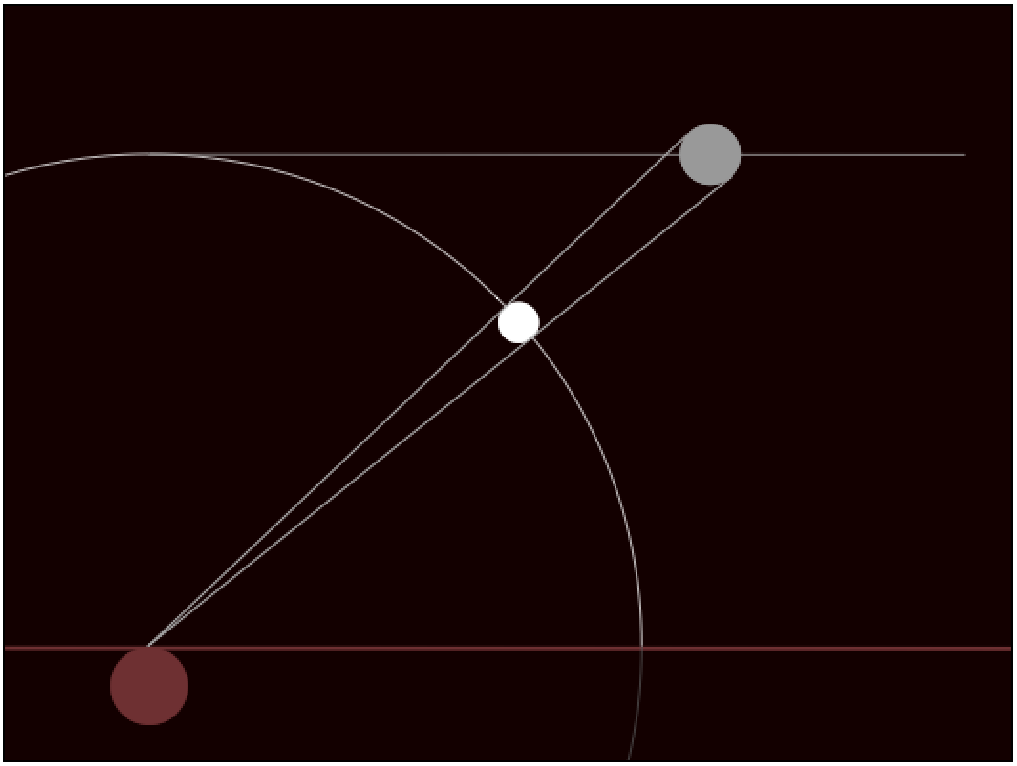
After passing through a second equal part of its orbit, the perceived moon in gray will lie farther above and beyond the real moon and will have increased a bit in perceived size.



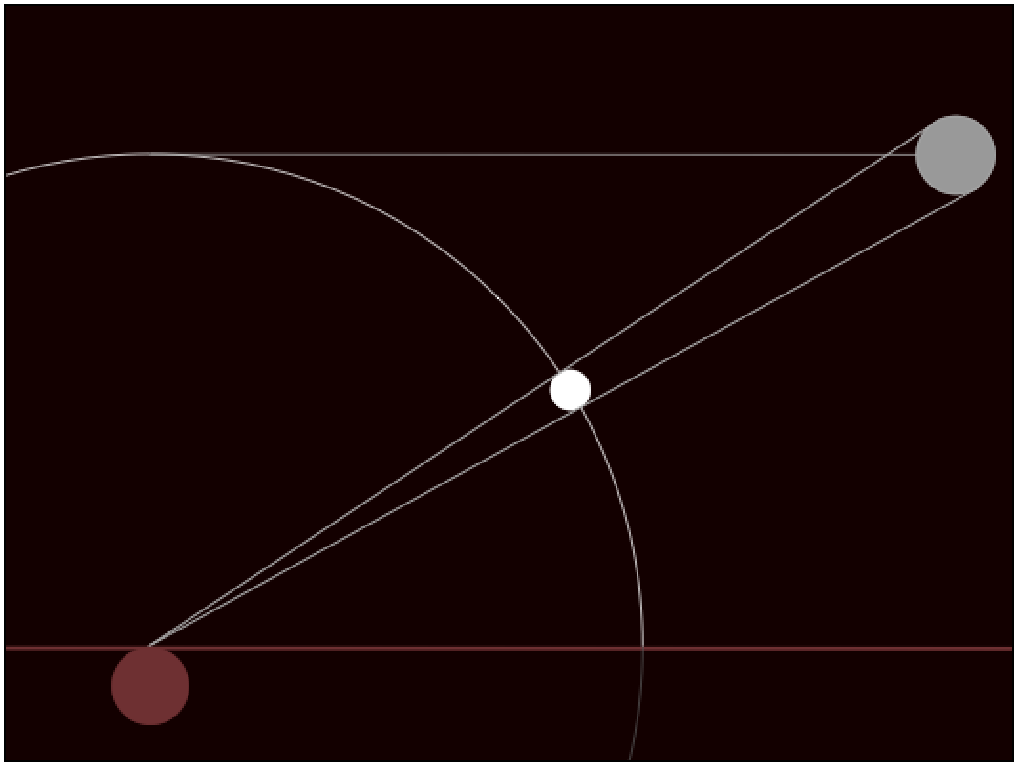
By this time the disparity in size between perceived and real moon will be noticeable,…



…and it will increase with each…



…increment of movement along its orbit.



Consequently, the closer it approaches the horizon on its actual orbit, the larger it *appears* to be because of a misperception of distance. Being parallel, the apparent celestial plane and the plane of the horizon will appear eventually to converge, as in this picture,…

…so when the Moon reaches that point at the horizon, it will have achieved its largest apparent size, as illustrated here.

This, in all its simplicity, elegance, and empirical plausibility, is Ibn al-Haytham's explanation of the Moon illusion. And it was precisely because of its elegance and plausibility that this explanation, with various adjustments, was generally accepted as canonical by elite scientific thinkers until relatively recent times.

Let me sum up my discussion with a few brief, concluding remarks. First, the theory of size and distance perception on which Ibn al-Haytham based his account of the Moon Illusion is neither counterintuitive nor simplistic. On the contrary, it makes a great deal of experiential and theoretical sense, and it's quite sophisticated. More to the point, within the intellectual context of its day, Ibn al-Haytham's account was strikingly original and creative. It's important to realize, however, that it was a response to issues of distance perception raised by his adaptation of Ptolemy's visual ray-theory.

Second, as Ibn al-Haytham analyzes it, such perception requires a great deal of psychological intervention on the part of the perceiver. Remember that, according to his theory, we don't perceive the external world directly or immediately. We do so by means of a visual representation standing between that world and us, as perceivers. Furthermore, this representation is two-dimensional, not three-dimensional, so it has no depth. Therefore, our perception of space, which is contingent on our perception of such characteristics as size and distance—both of which depend on depth-perception—is mediate and inferential, not immediate and intuitive. As we have seen, in fact, it is the product of deductive judgments based on a variety of clues contained in the flat visual representation.

Third, these deductive judgments are not intellectual; they are based in the imagination, which is where we "see" the mental images of size and distance that we've created over the course of our lives. That is why so much of spatial perception involves *mis*perception, because the imagination is so notoriously unreliable. And that is why such misperception requires intellectual rectification through the science of mathematical optics, which occupies most of the *Kitāb al-Manāẓir*. Yet even though we may be *intellectually* persuaded that the Moon Illusion is just that—an illusion—our intellectual awareness of this fact will not stop us from *perceiving* the moon as larger at horizon than at zenith. Perception, in short, is essentially autonomous. It's tied to what the visual representation tells us rather than to what our intellects tell us. Otherwise, we'd be able to reason our way out of seeing ourselves in mirrors because mirror-images are nothing but illusions, fictions of our imagination according to Ibn al-Haytham.

Fourth, and finally, implicit in Ibn al-Haytham's account of distance and size perception is the idea that space, as visually perceived, is a sort of fictional construct, a product of our imaginations. After all, as we've seen, Ibn al-Haytham assumes that spatial perception is ultimately based on how we sense space according to our bodies: the feeling of "an arm's length distant," "an arm-span tall," or "a pace away." How, then, do the two forms of spatial perception, tactile and visual, relate? Do they correspond in some fundamental, one-to-one way, each somehow an accurate reflection of the other? And if so, precisely how *do* they reflect one another? Do they in fact represent real space—whatever that may be—in any meaningful way? Is space actually geometrical in structure, or is our imposition of geometry upon it for visual analysis a mere artifice? To my knowledge, these issues weren't raised explicitly before the seventeenth century, but it's clear that they lurk just beneath the surface of Ibn al-Haytham's account. In retrospect, therefore, we cannot help but be impressed by how astute that account was at both the psychological and philosophical level.

Thank you for your kind attention!